Unit 4 Title	**Estimated Time Frame**
Rational Functions and **Exponential and Logarithmic Functions**	**40 days** or **20 block days**

Big Idea (s)

Extend their previous understanding of polynomial functions to rational functions.
Identify the key features of the graphs of rational functions and learn methods of solving rational equations.
Build upon and extend understanding of integer exponents to exponential functions.
Compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change.

Essential Question(s)

How do you calculate functions defined as quotients of polynomials, and what are the key features of their graphs?
For what input values is the output value positive, negative, or 0?
What happens to the output when the input value gets very large in magnitude?
How do you use exponential functions to solve problems?
How do you use exponential functions to model situations and solve problems?
When does a function best model a situation?
What kinds of transformations will affect the asymptote of an exponential function?
Why can't you divide an annual interest rate by 4 to obtain a quarterly interest rate?

Standards for Mathematical Practice (MP.) - The practice standards in bold describe expertise to be intentionally developed in this unit.	**Kentucky Interdisciplinary Literacy Practices (KILP.) -** The practice standards in bold describe expertise to be intentionally developed in Mathematics.
MP.1. Make sense of problems and persevere in solving them. **MP.2. Reason abstractly and quantitatively.** **MP.3. Construct viable arguments and critique the reasoning of others.** **MP.4. Model with mathematics.** **MP.5. Use appropriate tools strategically.** MP.6. Attend to precision. **MP.7. Look for and make use of structure.** **MP.8. Look for and express regularity in repeated reasoning.**	KILP.1 Recognize that text is anything that communicates a message. KLIP.2 Employ, develop, and refine schema to understand and create text. KILP.3 View literacy experiences as transactional, interdisciplinary and transformational. KILP.4 Utilize receptive & expressive language arts to better understand self, others, and the world. KILP.5 Apply strategic practices, with scaffolding & then independently, to approach new literacy tasks. **KILP.6 Collaborate with others to create new meaning.** **KILP.7 Utilize digital resources to learn and share with others.** **KILP.8 Engage in specialized, discipline specific literacy practices.** **KILP.9 Apply high level cognitive processes to think deeply and critically about text.** KILP.10.Develop a literacy identity that promotes lifelong learning.

Common Preconceptions/Misconceptions
Have students use a *Think-Aloud* strategy to develop the meaning and usage of the phrase "parent function." Students should simplify problems and appropriately use previously used vocabulary, such as terms, coefficients, and degrees, as they describe their process. In addition, students should use prior knowledge to describe the meaning of parts of an expression, such as a particular term or coefficient, and explain the meaning of the entire expression. To eliminate misconceptions, plan to approximate solutions with technology.

KAS Standards	Considerations	Samples of Learning Intentions and Success Criteria
KY.HS.A.17 Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise. **a.** Solve rational equations written as proportions in one variable. **MP.3, MP.5, MP.7**	Students analyze solution sets of equations to determine processes (for example, squaring both sides of an equation) that might lead to a solution set that differs from the original equation.	We are learning to solve a rational equation written as a proportion in one variable. **(Lesson 4-5)** ● I can solve a rational equation in one variable and identify extraneous solutions.
KY.HS.F.1 Understand properties and key features of functions and the different ways functions can be represented. **c.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. **d.** For an **exponential function** that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. **MP.2, MP.4, MP.7**	c. A function is often described and understood in terms of the output behavior, or over what input values is it increasing, decreasing, or constant. Important questions include, "For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?" Graphs become useful representations for understanding and comparing functions because these behaviors are often easy to see in the graphs of functions. Key features include, but are not limited to: intercepts; intervals where the function is increasing, decreasing, or remaining constant; relative maxima and minima; symmetries; end behavior; periodicity.	We are learning to identify and interpret key features of exponential functions. **(Lesson 6-1, 6-2)** ● I can sketch functions that model key feature behavior. ● I can describe key features of exponential functions.

KY.HS.F.4 Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★ **d.** Graph exponential and logarithmic functions, showing intercepts and end behavior.	Within a family, the functions often have commonalities in the shapes of their graphs and in the kinds of features important for identifying and describing functions. This standard indicates the function families in students' repertoires, detailing which features are required for several key families. Students demonstrate fluency with linear, quadratic and exponential functions, including the ability to graph without using technology. In other function families, students graph simple cases without technology and more complex ones with technology.	We are learning to graph exponential and logarithmic functions. **(Lesson 6-4)** ● I can use key features to graph exponential and logarithmic functions.
KY.HS.F.10 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents using technology. **MP.1, MP.7**	The inverse relationship between logarithmic and exponential functions is special in that each function's inverse is also a function.	We are learning to use inverses of logarithmic and exponential equations to solve those equations. **(Lesson 6-3, 6-4)** ● I can find equations of the inverses of exponential and logarithmic functions. ● I can evaluate and simplify logarithms. ● I can use logarithms to solve exponential models.

Supporting Standards

KY.HS. F. 5. b. Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. **MP.3, MP.6 (Lesson 6.2)**

KY.HS.F.4 Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). **This standard is a supporting standard for Logarithmic Functions for all students. **(Lesson 6.3)**
g. ★ (+) Advanced A2 students will graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. This standard should be assessed through modeling. **(Lesson 4.1)**

KY.HS.F.7 c. ★ (+) Translate between recursive and explicit formulas. **MP.4, MP.8, (Lesson 6.7)**

KY.HS.F.8 Understand the effects of transformations on the graph of a rational function. **MP.3, MP.5**
a. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); **and** find the value of k, given the graphs.
b. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
(Lessons 4.1)

KY.HS.F.9 Find inverse functions.
a. Given the equation of an invertible function, find the inverse.
b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2, MP.6**

KY.HS.F.12 Construct **exponential functions**, including ***geometric sequences***, given a graph, given a description of a relationship, or two input-output pairs (including reading these from a table).
MP.7, MP.8, (Lesson 6.7)

KY.HS.F.13 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **MP.7, MP.8**

KY.HS.F.14 Interpret the parameters in an exponential function in terms of a context. **MP.1, MP.2 (Lesson 6.1)**

KY.HS.A.4 (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. ★ **MP.1, MP. 4**

KY.HS.A.10 (+) Rewrite simple rational expressions in different forms. **MP.7, MP.8**
Advanced A2 students observe how to write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x) and r(x) are polynomials with the degree of r(x) less than the degree of b(x). **(Lessons 4.3 and 4.4)**
Methods of rewriting rational expressions could include but are not limited to

- Inspection
- Synthetic division
- Long division
- Use of technology

KY.HS.A.11 (+) Add, subtract, multiply and divide rational algebraic expressions. **MP.2, MP.3 (Lessons 4.3 and 4.4)**
Advanced A2 students go beyond demonstrating procedural fluency and apply this standard in various contextual situations.

KY.HS.A.12 Create equations and inequalities in one variable and use them to solve problems. **MP.1, MP.4**

KY.HS.A.13 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **MP.2, MP.5 (Lessons 4.1 and 4.5)**

KY.HS.A.16 Understand each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **MP.1, MP.3, (Lessons 4.5)**

Essential Vocabulary

Asymptote
Rational expression
Simplified form of a rational expression
Compound fraction
Extraneous solution
Rational equation

Common Assessment

Common Assessment Unit 4 Algebra 2

Anchor Resources

enVision Topic 4 Rational Functions	enVision Topic 6 Exponential and Logarithmic Functions
MILC - Rational Functions Resources in MILC **FAL** (one per semester): **Representing Linear and Exponential Growth** (After 6-2) ****Recommended FAL for 2nd semester)** **** (recommend) 3 Act Math Task** - Real Cool Waters (from enVision) (anytime after Lesson 4.5)	**MILC - Exponential and Logarithmic Function Resources** **3 Act Math Task -** The Crazy Conditioning (Use after Lesson 6.2) ** Anytime FAL - **Evaluating Statements About Radicals** (use with 5-4 OR anytime after 5-4 for review for ACT and more)

*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

** Mathematical Practices (A.MP.1- 8) should be evidenced throughout each unit, depending on the tasks.

*** Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.