Activity

6-1 Key Features of Exponential Functions

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EXPLORE & REASON

Margaret investigates three functions: $y = 3x$, $y = x^3$, and $y = 3^x$. She is interested in the differences and ratios between consecutive y-values. Here is the table she started for $y = 3x$.

Investigating $y = 3x$			
x	y	Difference between y-values	Ratio between y-values
1	3		
2	6	$6 - 3 = 3$	$\frac{6}{3} = 2$
3	9	$9 - 6 = 3$	$\frac{9}{6} = 1.5$
4	12	$12 - 9 = 3$	$\frac{12}{9} \approx 1.33$

A. Create tables like Margaret's for all three functions and fill in more rows.

B. Which functions have a constant difference between consecutive y-values? Constant ratio?

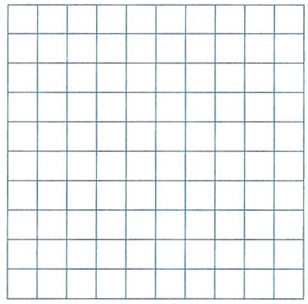
C. **Use Structure** Which of these three functions will have y-values that increase the fastest as x increases? Why?

HABITS OF MIND

Generalize Let b represent a whole number. For $b > 1$, which function do you think will increase at a faster rate as x increases, $f(x) = b^x$ or $g(x) = x^b$? Explain.

EXAMPLE 1 Try It! Identify Key Features of Exponential Functions

1. Graph $f(x) = 4(0.5)^x$. What are the domain, range, intercepts, asymptote, and the end behavior for this function?



EXAMPLE 2 Try It! Graph Transformations of Exponential Functions

2. How do the asymptote and intercept of the given function compare to the asymptote and intercept of the function $f(x) = 5^x$?

   a. $g(x) = 5^{x+3}$

   b. $h(x) = 5^{-x}$

HABITS OF MIND

Reason What kinds of transformations will affect the asymptote or the intercept(s) of an exponential function? Explain.

EXAMPLE 3 Try It! Model with Exponential Functions

3. A factory purchased a 3D Printer on January 2, 2010. The value of the printer is modeled by the function $f(x) = 30(0.93)^x$, where x is the number of years since 2010.
   a. What is the value of the printer after 10 years?
   b. Does the printer lose more of its value in the first 10 years or in the second?

EXAMPLE 4 Try It! Interpret an Exponential Function

4. Two-hundred twenty hawks were released into a region in 2016. The function $f(x) = 220(1.05)^x$ can be used to model the number of red-tailed hawks in the region x years after 2016.
   a. Is the population increasing or decreasing? Explain.
   b. In what year will the number of hawks reach 280?

HABITS OF MIND

Use Structure How can you determine the growth or decay factor by looking at an exponential function? The growth or decay rate?

EXAMPLE 5 Try It! Compare Two Exponential Functions

5. In Example 5, will the value of the painting ever surpass the value of the sculpture according to the models? Explain.

HABITS OF MIND

Reason For two functions $f(x) = b^x$ and $g(x) = b^{x+n}$, where $n > 0$, is it possible that the two graphs will intersect? Explain.

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do graphs and equations reveal key features of exponential growth and decay functions?

2. **Vocabulary** How do *exponential functions* differ from polynomial and rational functions?

3. **Error Analysis** Charles claimed the function $f(x) = \left(\frac{3}{2}\right)^x$ represents exponential decay. Explain the error Charles made.

4. **Communicate Precisely** How are exponential growth functions similar to exponential decay functions? How are they different?

Do You KNOW HOW?

5. Graph the function $f(x) = 4 \times 3^x$. Identify the domain, range, intercept, asymptote, and describe the end behavior.

6. The exponential function $f(x) = 2500(0.4)^x$ models the amount of money in Zachary's savings account over the last 10 years. Is Zachary's account balance increasing or decreasing? Write the base in terms of the rate of growth or decay.

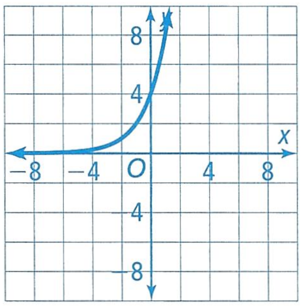
7. Describe how the graph of $g(x) = 4(0.5)^{x-3}$ compares to the graph of $f(x) = 4(0.5)^x$.

8. Two trucks were purchased by a landscaping company in 2016. Their values are modeled by the functions $f(x) = 35(0.85)^x$ and $g(x) = 46(0.75)^x$ where x is the number of years since 2016. Which function models the truck that is worth the most after 5 years? Explain.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

9. Use Structure What value of a completes the equation $y = a \cdot 2^x$ for the exponential growth function shown below?



10. Make Sense and Persevere Cindy found a collection of baseball cards in her attic worth \$8,000. The collection is estimated to increase in value by 1.5% per year. Write an exponential growth function and find the value of the collection after 7 years.

11. Error Analysis Describe and correct the error a student made in identifying the growth or decay factor for the function $y = 2.55(0.7)^x$.

Step 1 The base of the function is 0.7, so it represents exponential decay.

Step 2 The function in the form $y = a(1 - r)^x$ is $y = 2.55(1 - 0.7)^x$.

Step 3 The decay factor is 0.3.

12. Reason In 2000, the population of St. Louis was 346,904, and it decreased to 319,257 in 2010. If this population decrease were modeled by an exponential decay function, what value would represent the y-intercept? Explain your reasoning.

13. Mathematical Connections Describe how the graph of $g(x) = 6 \cdot 2^{x+1} - 4$ compares to the graph of $f(x) = 6 \cdot 2^x$.

PRACTICE & PROBLEM SOLVING

PRACTICE

Identify the domain, range, intercept, and asymptote of each exponential function. Then describe the end behavior. SEE EXAMPLE 1

14. $f(x) = 5 \cdot 3^x$

15. $f(x) = 0.75\left(\frac{2}{3}\right)^x$

16. $f(x) = 4\left(\frac{1}{2}\right)^x$

17. $f(x) = 7 \cdot 2^x$

Determine whether each function represents exponential growth or decay. Write the base in terms of the rate of growth or decay, identify *r*, and interpret the rate of growth or decay.
SEE EXAMPLES 3 AND 4

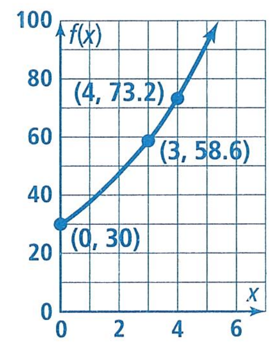
18. $y = 100 \cdot 2.5^x$

19. $f(x) = 10{,}200\left(\frac{3}{5}\right)^x$

20. $f(x) = 12{,}000\left(\frac{7}{10}\right)^x$

21. $y = 450 \cdot 2^x$

22. The function $f(x)$, shown in the graph, represents an exponential growth function. Compare the average rate of change of $f(x)$ to the average rate of change of the exponential growth function $g(x) = 25\,(1.4)^x$. Use the interval [0, 4]. SEE EXAMPLE 5



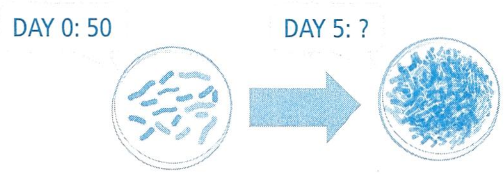
23. Write a function $g(x)$ that represents the exponential function $f(x) = 2^x$ after a vertical stretch of 6 and a reflection across the x-axis. Graph both functions. SEE EXAMPLE 2

24. The population of Medway, Ohio, was 4,007 in 2000. It is expected to decrease by about 0.36% per year. Write an exponential decay function and use it to approximate the population in 2020. SEE EXAMPLE 4

PRACTICE & PROBLEM SOLVING

APPLY

25. Model With Mathematics A colony of bacteria starts with 50 organisms and quadruples each day. Write an exponential function, $P(t)$, that represents the population of the bacteria after t days. Then find the number of bacteria that will be in the colony after 5 days.



26. Higher Order Thinking The number of teams y remaining in a single elimination tournament can be found using the exponential function $y = 128\left(\frac{1}{2}\right)^x$, where x is the number of rounds played in the tournament.

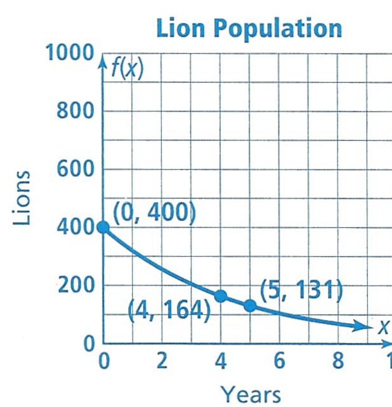
a. Determine whether the function represents exponential growth or decay. Explain.

b. What does 128 represent in the function?

c. What percent of the teams are eliminated after each round? Explain how you know.

d. Graph the function. What is a reasonable domain and range for the function? Explain.

27. Construct Arguments The function shown in the graph represents the number of lions in a region after x years, where the rate of decay is 20%. The number of zebras in that same region after x years can be modeled by the function $f(x) = 300(0.95)^x$. A representative for a conservationist group claims there will be fewer lions than zebras within 2 years. Is the representative correct? Justify your answer.

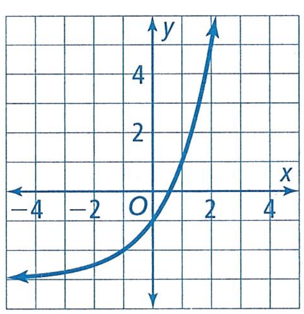


ASSESSMENT PRACTICE

28. The exponential function $g(x) = 3^{x-1} + 6$ is a transformation of the function $f(x) = 3^x$. Does each statement accurately describe how the graph of $g(x)$ compares to the graph of $f(x)$? Select yes or no.

	Yes	No
a. $g(x)$ is translated 6 units up.	❑	❑
b. $g(x)$ is translated 6 units down.	❑	❑
c. $g(x)$ is translated 6 units to the right.	❑	❑
d. $g(x)$ is translated 1 unit to the right.	❑	❑
e. $g(x)$ is translated 1 unit to the left.	❑	❑
f. The horizontal asymptote shifts 1 unit down.	❑	❑

29. SAT/ACT Which of the functions defined below could be the one shown in this graph?



Ⓐ $f(x) = 4(2)^{x-1} + 3$

Ⓑ $f(x) = 4(2)^{x+1} + 3$

Ⓒ $f(x) = 4(2)^{x-1} - 3$

Ⓓ $f(x) = 4(2)^{x+1} - 3$

30. Performance Task A radioactive isotope of the element osmium Os-182 has a half-life of 21.5 hours. This means that if there are 100 grams of Os-182 in a sample, after 21.5 hours there will only be 50 grams of that isotope remaining.

Part A Write an exponential decay function to model the amount of Os-182 in a sample over time. Use A_0 for the initial amount and A for the amount after time t in hours.

Part B Use your model to predict how long it would take a sample containing 500 g of Os-182 to decay to the point where it contained only 5 g of Os-182.

6-2

Exponential Models

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EXPLORE & REASON

Juan is studying exponential growth of bacteria cultures. Each is carefully controlled to maintain a specific growth rate. Copy and complete the table to find the number of bacteria cells in each culture.

Culture	Initial Number of Bacteria	Growth Rate per Day	Time (days)	Final Number of Bacteria
A	10,000	8%	1	
B	10,000	4%	2	
C	10,000	2%	4	
D	10,000	1%	8	

A. What is the relationship between the daily growth rate and the time in days for each culture?

B. **Look for Relationships** Would you expect a culture with a growth rate of $\frac{1}{2}$% and a time of 16 days to have more or fewer cells than the others in the table? Explain.

HABITS OF MIND

Model With Mathematics Describe another situation that you could represent using an exponential function.

Notes

Assess

EXAMPLE 1

Try It! Rewrite an Exponential Function to Identify a Rate

1. The population in a small town is increasing annually at 1.8%. What is the quarterly rate of population increase?

HABITS OF MIND

Generalize Why can't you just divide an annual interest rate by 4 to obtain a quarterly interest rate?

EXAMPLE 2

Try It! Understand Continuously Compounded Interest

2. $3,000 is invested in an account that earns 3% annual interest, compounded monthly.
   a. What is the value of the account after 10 years?
   b. What is the value of the account after 100 years?

EXAMPLE 3

Try It! Understanding Continuously Compounded Interest

3. If you continued the table for $n = 1{,}000{,}000$, would the value in the account increase or decrease? How do you know?

HABITS OF MIND

Generalize Which yields the greatest return on investment: compounding quarterly, hourly, or continuously? Explain.

Assess

EXAMPLE 4

Try It! Find Continuously Compounded Interest

4. You invest $125,000 in an account that earns 4.75% annual interest, compounded continuously.

   a. What is the value of the account after 15 years?

   b. What is the value of the account after 30 years?

EXAMPLE 5

Try It! Use Two Points to Find an Exponential Model

5. A surveyor determined the value of an area of land over a period of several years since 1950. The land was worth $31,000 in 1954 and $35,000 in 1955. Use the data to determine an exponential model that describes the value of the land.

EXAMPLE 6

Try It! Use Regression to Find an Exponential Model

6. According to the model in Example 6, what was the approximate temperature 35 minutes after cooling started?

HABITS OF MIND

Generalize How can a graph help you determine whether an exponential model is appropriate for a data set? Explain.

Do You UNDERSTAND?

1. ESSENTIAL QUESTION Why do you develop exponential models to represent and interpret situations?

2. **Error Analysis** The exponential model $y = 5{,}000(1.05)^t$ represents the amount Yori earns in an account after t years when \$5,000 is invested. Yori said the monthly interest rate of the exponential model is 5%. Explain Yori's error.

3. **Vocabulary** Explain the similarities and differences between compound interest and continuously compounded interest.

4. **Communicate Precisely** Kylee is using a calculator to find an exponential regression model. How would you explain to Kylee what the variables in the model $y = ab^x$ represent?

Do You KNOW HOW?

The exponential function models the annual rate of increase. Find the monthly and quarterly rates.

5. $f(t) = 2{,}000(1.03)^t$

6. $f(t) = 500(1.055)^t$

Find the total amount of money invested in an account at the end of the given time period.

7. compounded monthly, $P = \$2{,}000$, $r = 3\%$, $t = 5$ years

8. continuously compounded, $P = \$1{,}500$, $r = 1.5\%$, $t = 6$ years

Write an exponential model given two points.

9. (3, 55) and (4, 70)

10. (7, 12) and (8, 25)

11. Paul invests \$6,450 in an account that earns continuously compounded interest at an annual rate of 2.8%. What is the value of the account after 8 years?

PRACTICE & PROBLEM SOLVING

UNDERSTAND

12. **Error Analysis** Suppose $6,500 is invested in an account that earns interest at a rate of 2% compounded quarterly for 10 years. Describe and correct the error a student made when finding the value of the account.

$$A = 6500\left(1 + \frac{0.02}{12}\right)^{12(10)}$$

$$A = 7937.80$$

✗

13. **Communicate Precisely** The points (2, 54.61) and (4, 403.48) are points on the graph of an exponential model in the form $y = a \cdot e^x$.

a. Explain how to write the exponential model, and then write the model.

b. How can you use the exponential model to find the value of y when $x = 8$?

14. **Model with Mathematics** Use the points listed in the table for years 7 and 8 to find an exponential model. Then use a calculator to find an exponential model for the data. Explain how to find each model. Predict the amount in the account after 15 years.

Time (yr)	Amount ($)
1	3,225
2	3,500
3	3,754
4	4,042
5	4,368
6	4,702
7	5,063
8	5,456

15. **Higher Order Thinking** A power model is a type of function in the form $y = a \cdot x^b$. Use the points (1, 4), (2, 8), (3, 16) and (4, 64) and a calculator to find an exponential model and a power model for the data. Then use each model to predict the value of y when $x = 6$. Graph the points and models in the same window. What do you notice?

PRACTICE & PROBLEM SOLVING

PRACTICE

Find the amount in the account for the given principal, interest rate, time, and compounding period. SEE EXAMPLES 2 AND 4

16. $P = 800$, $r = 6\%$, $t = 9$ years; compounded quarterly

17. $P = 3{,}750$, $r = 3.5\%$, $t = 20$ years; compounded monthly

18. $P = 2{,}400$, $r = 5.25\%$, $t = 12$ years; compounded semi-annually

19. $P = 1{,}500$, $r = 4.5\%$, $t = 3$ years; compounded daily

20. $P = \$1{,}000$, $r = 2.8\%$, $t = 5$ years; compounded continuously

21. $P = \$16{,}000$, $r = 4\%$, $t = 25$ years; compounded continuously

Write an exponential model given two points.
SEE EXAMPLE 5

22. (9, 140) and (10, 250)

23. (6, 85) and (7, 92)

24. (10, 43) and (11, 67)

25. In 2012, the population of a small town was 3,560. The population is decreasing at a rate of 1.7% per year. How can you rewrite an exponential growth function to find the quarterly decay rate? SEE EXAMPLE 1

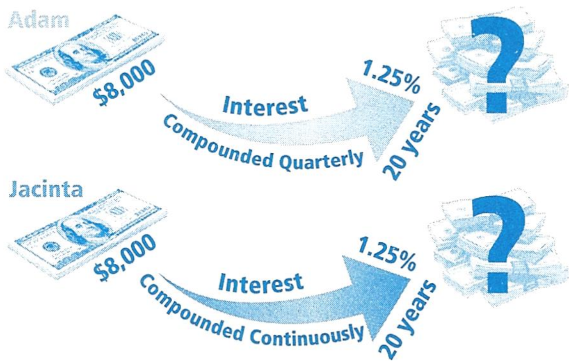
26. Selena took a pizza out of the oven and it started to cool to room temperature (68°F). She will serve the pizza when it reaches 150°F. She took the pizza out of the oven at 5:00 P.M. When can she serve it? SEE EXAMPLE 6

Time (min)	Temperature (°F)
5	310
8	264
10	238
15	202
20	186
25	175

PRACTICE & PROBLEM SOLVING

APPLY

27. Reason Adam invests $8,000 in an account that earns 1.25% interest, compounded quarterly for 20 years. On the same date, Jacinta invests $8,000 in an account that earns continuous compounded interest at a rate of 1.25% for 20 years. Who do you predict will have more money in their account after 20 years? Explain your reasoning.



28. Make Sense and Persevere A blogger found that the number of visits to her Web site increases 5.6% annually. The Web site had 80,000 visits this year. Write an exponential model to represent this situation. By what percent does the number of visits increase daily? Explain how you found the daily rate.

29. Use Structure Jae invested $3,500 at a rate of 2.25% compounded continuously in 2010. How much will be in the account in 2025? How much interest will the account have earned by 2025?

30. Model with Mathematics A scientist is conducting an experiment with a pesticide. Use a calculator to find an exponential model for the data in the table. Use the model to determine how much pesticide remains after 180 days.

Day 0	20.00g
Day 1	14.73g
Day 2	11.29g
Day 3	8.38g
Day 4	6.82g
Day 5	4.75g
Day 6	3.15g

ASSESSMENT PRACTICE

31. The table shows the account information of five investors. Which of the following are true, assuming no withdrawals are made? Select all that apply.

Employee	P	r	t(years)	Compound
Anna	4000	1.5%	12	Quarterly
Nick	2500	3%	8	Monthly
Lori	7200	5%	15	Annually
Tara	2100	4.5%	6	Continuously
Steve	3800	3.5%	20	Semi-annually

Ⓐ After 12 years, Anna will have about $4,788.33 in her account.

Ⓑ After 8 years, Nick will have about $3,177.17 in his account.

Ⓒ After 15 years, Lori will have about $15,218.67 in her account.

Ⓓ After 6 years, Tara will have about $2,750.93 in her account.

Ⓔ After 20 years, Steve will have about $7,629.00 in his account.

32. SAT/ACT Rick invested money in a continuous compound account with an interest rate of 3%. How long will it take Rick's account to double?

Ⓐ about 2 years

Ⓑ about 10 years

Ⓒ about 23 years

Ⓓ about 46 years

Ⓔ about 67 years

33. Performance Task Cassie is financing a $2,400 treadmill. She is going to use her credit card for the purchase. Her card charges 17.5% interest compounded monthly. She is not required to make minimum monthly payments.

Part A How much will Cassie pay in interest if she waits a full year before paying the full balance?

Part B How much additional interest will Cassie pay if she waits two full years before paying the full balance?

Part C If both answers represent a single year of interest, why is the answer in B greater than the answer in A?

Video

MATHEMATICAL MODELING IN 3 ACTS

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The Crazy Conditioning

Like all sports, soccer requires its players to be well trained. That is why players often have to run sprints in practice.

To make sprint drills more interesting, many coaches set up competitions. Coaches might split the players into teams and have them run relay races against each other. Or they might have the players sprint around cones and over barriers. What other ways would make doing sprints more fun? Think about this during this Mathematical Modeling in 3 Acts lesson.

ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?

2. Write down the main question you will answer about the video.

3. Make an initial conjecture that answers this main question.

4. Explain how you arrived at your conjecture.

5. Write a number that you know is too small.

6. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

ACT 2 Develop a Model

7. Use the math that you have learned in the topic to refine your conjecture.

8. Is your refined conjecture between the high and low estimates you came up with earlier?

ACT 3 Interpret the Results

9. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

6-3 Logarithms

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CRITIQUE & EXPLAIN

Earthquakes make seismic waves through the ground. The equation $y = 10^x$ relates the height, or amplitude, in microns, of a seismic wave, y, and the power, or magnitude, x, of the ground-shaking it can cause.

Magnitude, x	Amplitude, y
2	100
3	1,000
?	4,500
4	10,000

Taylor and Chen used different methods to find the magnitude of the earthquake with amplitude 5,500.

Taylor

5,500 is halfway between 1,000 and 10,000.

3.5 is halfway between 3 and 4.

The magnitude is about 3.5.

Chen

$y = 10^x$

$10^3 = 1{,}000$
$10^4 = 10{,}000$
$10^{3.5} \approx 3{,}162$
$10^{3.7} \approx 5{,}012$
$10^{3.8} \approx 6{,}310$
$10^{3.74} \approx 5{,}500$

The magnitude is about 3.74.

A. What is the magnitude of an earthquake with amplitude 100,000? How do you know?

B. **Construct Arguments** Critique Taylor's and Chen's work. Is each method valid? Could either method be improved?

C. Describe how to express the exact value of the desired magnitude.

HABITS OF MIND

Reason Taylor reasoned that since 5,500 was halfway between 1,000 and 10,000, that the magnitude had to be halfway between 3 and 4. What is incorrect about Taylor's reasoning?

EXAMPLE 1

Try It! Understand Logarithms

1. Write the logarithmic form of $y = 8^x$.

EXAMPLE 2

Try It! Convert Between Exponential and Logarithmic Forms

2. a. What is the logarithmic form of $7^3 = 343$?

   b. What is the exponential form of $\log_4 16 = 2$?

HABITS OF MIND

Communicate Precisely Write a sentence to describe what the equation $\log_a b = c$ means.

EXAMPLE 3

Try It! Evaluate Logarithms

3. What is the value of each logarithmic expression?

   a. $\log_3 \left(\frac{1}{81}\right)$ b. $\log_7 (-7)$ c. $\log_5 5^9$

Notes

Assess

EXAMPLE 4

Try It! Evaluate Common and Natural Logarithms

4. What is the value of each logarithmic expression to the nearest ten-thousandth?

a. $\log 321$

b. $\ln 1{,}215$

c. $\log 0.17$

HABITS OF MIND

Reason In order for $\log x$ or $\ln x$ to be defined, what must be true about x?

EXAMPLE 5

Try It! Solve Equations With Logarithms

5. Solve each equation. Round to the nearest thousandth.

a. $\log (3x - 2) = 2$

b. $e^{x+2} = 8$

EXAMPLE 6

Try It! Use Logarithms to Solve Equations

6. What is the magnitude of an earthquake with a seismic energy of 1.8×10^{23} joules?

HABITS OF MIND

Make Sense and Persevere How do logarithms help you to solve an equation in which the variable is an exponent?

Do You UNDERSTAND?

1. ESSENTIAL QUESTION What are logarithms and how are they evaluated?

2. **Error Analysis** Amir said the expression $\log_5(-25)$ simplifies to –2. Explain Amir's possible error.

3. **Vocabulary** Explain the difference between the common logarithm and the natural logarithm.

4. **Make Sense and Persevere** How can logarithms help to solve an equation such as $10^t = 656$?

Do You KNOW HOW?

Write each equation in logarithmic form.

5. $2^{-6} = \frac{1}{64}$

6. $e^4 \approx 54.6$

Write each equation in exponential form.

7. $\log 200 \approx 2.301$

8. $\ln 25 \approx 3.22$

Evaluate the expression.

9. $\log_4 64$

10. $\log \frac{1}{100}$

11. $\ln e^5$

12. Solve for x. $4e^x = 7$.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

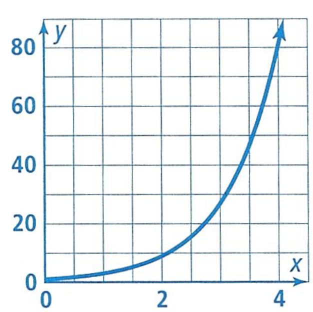
13. Make Sense and Persevere If the LN button on your calculator were broken, how could you still use your calculator to find the value of the expression ln 65?

14. Error Analysis Describe and correct the error a student made in solving an exponential equation.

$$16e^t = 98$$
$$e^t = 6.125$$
$$6.125t = \ln e$$
$$t = \frac{\ln e}{6.125}$$

✗

15. Higher Order Thinking Use the graph of $y = 3^x$ to estimate the value of $\log_3 50$. Explain your reasoning.



16. Generalize For what values of x is the expression $\log_4 x < 0$ true?

17. Use Structure A student says that $\log_3\left(\frac{1}{27}\right)$ simplifies to −3. Is the student correct? Explain.

18. Use Structure Explain why the expression ln 1,000 is not equal to 3.

PRACTICE & PROBLEM SOLVING

PRACTICE

Write the inverse of each exponential function.
SEE EXAMPLE 1

19. $y = 4^x$

20. $y = 10^x$

21. $y = 7^x$

22. $y = a^x$

Write each equation in logarithmic form.
SEE EXAMPLE 2

23. $3^8 = 6{,}561$

24. $e^{-3} \approx 0.0498$

25. $5^0 = 1$

26. $7^3 = 343$

Write each equation in exponential form.
SEE EXAMPLE 2

27. $\log\frac{1}{100} = -2$

28. $\log_8 64 = 2$

29. $\ln 148.41 \approx 5$

30. $\log_2 \frac{1}{32} = -5$

Evaluate each logarithmic expression. SEE EXAMPLE 3

31. $\log_5 \frac{1}{125}$

32. $\log_6 (-216)$

33. $\log_3 3^4$

34. $\log_2 32$

35. $\log_9 729$

36. $\log_8 \frac{1}{64}$

37. $\log_7 0$

38. $\log_7 7^a$

Use a calculator to evaluate each expression. Round to the nearest ten-thousandth. SEE EXAMPLE 4

39. $\log 78.5$

40. $\log 0.24$

41. $\ln(-37)$

42. $\ln 41.5$

43. $\log 12$

44. $\ln 3$

Solve each equation. Round answers to the nearest ten-thousandth. SEE EXAMPLES 5 AND 6

45. $\log(7x + 6) = 3$

46. $2.75e^t = 38.6$

47. $\ln(3x - 1) = 2$

48. $10^{t+1} = 50$

49. $1.5e^t = 27$

50. $\log(x - 3) = -1$

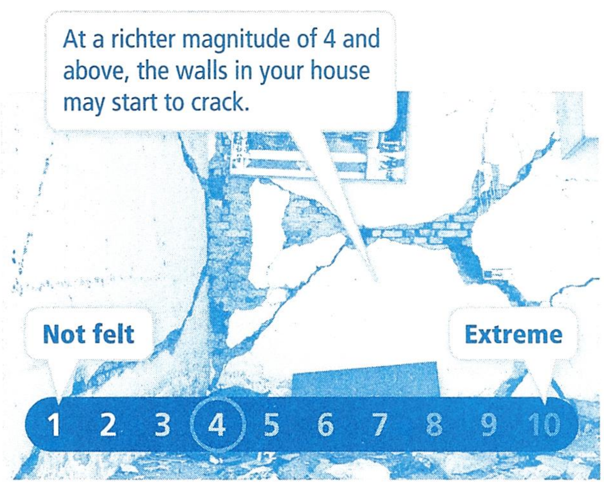
51. How long does it take for \$250 to grow to \$600 at 4% annual percentage rate compounded continuously? Round to the nearest year.

PRACTICE & PROBLEM SOLVING

APPLY

52. **Model with Mathematics** Michael invests \$1,000 in an account that earns a 4.75% annual percentage rate compounded continuously. Peter invests \$1,200 in an account that earns a 4.25% annual percentage rate compounded continuously. Which person's account will grow to \$1,800 first?

53. **Reason** The Richter magnitude of an earthquake is $R = 0.67\log(0.37E) + 1.46$, where E is the energy (in kilowatt-hours) released by the earthquake.



a. What is the magnitude of an earthquake that releases 11,800,000,000 kilowatt-hours of energy? Round to the nearest tenth.

b. How many kilowatt-hours of energy would an earthquake have to release in order to be an 8.2 on the Richter scale? Round to the nearest whole number.

c. What number of kilowatt-hours of energy would an earthquake have to release in order for walls to crack? Round to the nearest whole number.

54. **Reason** The function $c(t) = 108e^{-0.08t} + 75$ calculates the temperature, in degrees Fahrenheit, of a cup of coffee that was handed out a drive-thru window t minutes ago.

a. What is the temperature of the coffee in the instant that it is handed out the window?

b. After how many minutes is the coffee in the cup 98 degrees Fahrenheit? Round to the nearest whole minute.

ASSESSMENT PRACTICE

55. Given that $\log_b x < 0$, which of the following are true? Select all that apply.

Ⓐ $b < 0$

Ⓑ $x < 0$

Ⓒ $b > 0$

Ⓓ $x > 0$

Ⓔ $x < 1$

56. **SAT/ACT** In the equation $\log_3 a = b$, if b is a whole number, which of the following CANNOT be a value for a?

Ⓐ 1

Ⓑ 3

Ⓒ 6

Ⓓ 9

Ⓔ 81

57. **Performance Task** Money is deposited into two separate accounts. The money in one account is compounded continuously. The money in the other account is not compounded continuously. Neither account has any money withdrawn in the first 6 years.

Year	Account 1 Balance ($)	Account 2 Balance ($)
0	400	500
1	433.31	575
2	469.40	650
3	508.50	725
4	550.85	800
5	596.72	875

Part A Write a function to calculate the amount of money in each account given t, the number of years since the account was opened. Describe the growth in each account.

Part B Will the amount of money in Account 1 ever exceed the amount of money in Account 2? Explain. If so, when will that occur?

Activity

6-4 Logarithmic Functions

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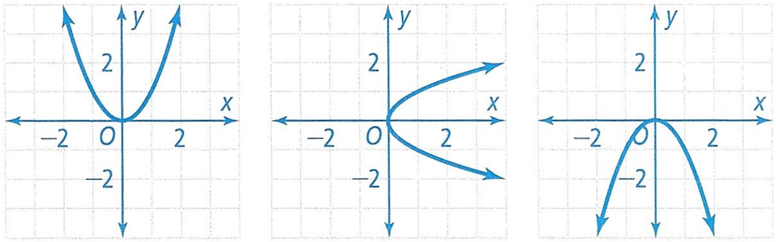
EXPLORE & REASON

Compare the graphs.



A. Which two graphs represent the inverse of each other? Explain.

B. Look for Relationships What is the relationship between the domain and the range of the two inverse relations?

HABITS OF MIND

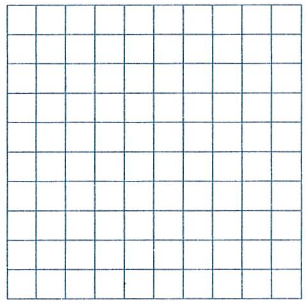
Communicate Precisely How are the points on graphs of functions that are inverses of each other related?

✎ Notes

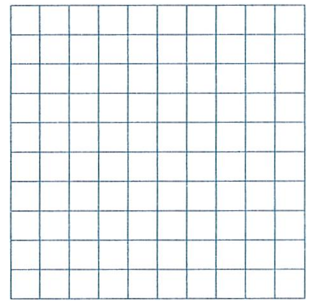
EXAMPLE 1 **Try It!** **Identify Key Features of Logarithmic Functions**

1. Graph each function and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior.

a. $y = \ln x$



b. $y = \log_{\frac{1}{2}} x$



EXAMPLE 2 **Try It!** **Graph Transformations of Logarithmic Functions**

2. Describe how each graph compares to the graph of $f(x) = \ln x$.

a. $g(x) = \ln x + 4$

b. $h(x) = 5 \ln x$

HABITS OF MIND

Use Structure Does the graph of either $y = \ln x + 4$ or $y = \ln (x + 4)$ have an intercept that is different from the intercept of $y = \ln x$? Explain.

EXAMPLE 3 Try It! Inverses of Exponential and Logarithmic Functions

3. Find the inverse of each function.

a. $f(x) = 3^{x+2}$

b. $g(x) = \log_7 x - 2$

EXAMPLE 4 Try It! Interpret the Inverse of a Formula Involving Logarithms

4. Describe what happens to the amount of monthly revenue as the amount of advertising increases. How might you determine the optimal advertising budget? Explain.

HABITS OF MIND

Generalize How would you explain, in your own words, how to find the inverse of a logarithmic function?

EXAMPLE 5 Try It! Compare Two Logarithmic Functions

5. For which plane do you think the altitude will change more quickly over the interval $15 \leq t \leq 20$? Explain your reasoning.

HABITS OF MIND

Look for Relationships How does the average rate of change of the function $f(x) = \log x$ change as x increases?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How is the relationship between logarithmic and exponential functions revealed in the key features of their graphs?

2. **Error Analysis** Raynard claims the domain of the function $y = \log_3 x$ is all real numbers. Explain the error Raynard made.

3. **Communicate Precisely** How are the graphs of $f(x) = \log_5 x$ and $g(x) = -\log_5 x$ related?

Do You KNOW HOW?

4. Graph the function $y = \log_4 x$ and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior.

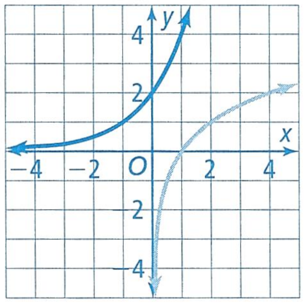
5. Write the equation for the function $g(x)$, which can be described as a vertical shift $1\frac{1}{2}$ units up from the function $f(x) = \ln x - 1$.

6. The function $y = 5 \ln (x + 1)$ gives y, the number of downloads, in hundreds, x minutes after the release of a song. Find the equation of the inverse and interpret its meaning.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

7. **Look for Relationships** Are the logarithmic and exponential functions shown inverses of each other? Explain.



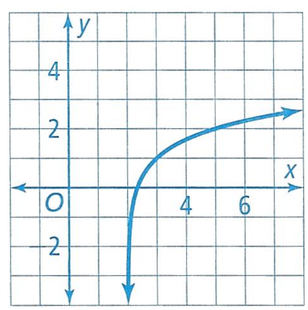
8. **Communicate Precisely** How is the graph of the logarithmic function $g(x) = \log_2 (x - 7)$ related to the graph of the function $f(x) = \log_2 x$? Explain your reasoning.

9. **Error Analysis** Describe and correct the error a student made in finding the inverse of the exponential function $f(x) = 5^{x-6} + 2$.

$y = 5^{x-6} + 2$	Write in $y = f(x)$ form.
$x = 5^{y-6} + 2$	Interchange x and y.
$x - 2 = 5^{y-6}$	Subtract 2 from each side.
$y - 6 = \log_5 x - 2$	Rewrite in logarithmic form.
$y = \log_5 x - 2 + 6$	Add 6 to each side.
$y = \log_5 x + 4$	Simplify.
$f'(x) = \log_5 x + 4$	

10. **Make Sense and Persevere** The number of members m who joined a new workout center w weeks after opening is modeled by the equation $m = 1.6^{w+2}$, where $0 \le w \le 10$. Find the inverse of the function and explain what the inverse tells you.

11. **Use Structure** The graph shows a transformation of the parent graph $f(x) = \log_3 x$. Write an equation for the graph.



PRACTICE & PROBLEM SOLVING

PRACTICE

Graph each function and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior. SEE EXAMPLE 1

12. $y = \log_5 x$

13. $y = \log_8 x$

14. $y = \log_{\frac{3}{10}} x$

15. $y = \log_{0.1} x$

Describe the graph in terms of transformations of the parent function $f(x) = \log_6 x$. Compare the asymptote and x-intercept of the given function to the parent function. SEE EXAMPLE 2

16. $g(x) = \frac{1}{2} \log_6 x$

17. $g(x) = \log_6 (-x)$

18. Describe how the graph of $g(x) = -\ln(x + 0.5)$ is related to the graph of $f(x) = \ln x$. SEE EXAMPLE 2

Find the equation of the inverse of each function. SEE EXAMPLE 3

19. $f(x) = 5^{x-3}$

20. $f(x) = \left(\frac{1}{2}\right)^{x-1}$

21. $f(x) = 6^{x+7}$

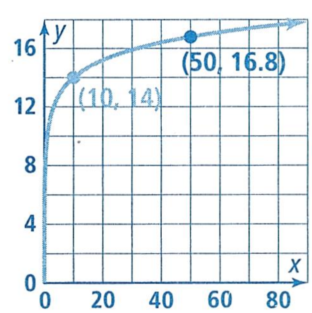
22. $f(x) = \log_2 (8x)$

23. $f(x) = \ln (x + 3) - 1$

24. $f(x) = 4 \log_2 (x - 3) + 2$

25. The altitude y, in feet, of a plane t minutes after takeoff is approximated by the function $y = 5{,}000 \ln(.05t) + 8{,}000$. Solve for t in terms of y. What is a situation in which it would be easier to use your new equation rather than the original? SEE EXAMPLE 4

26. Find the average rate of change of the function graphed below over the interval $10 \leq x \leq 50$. Compare it to the average rate of change of $y = 3 \log x + 12$ over the same interval. SEE EXAMPLE 5



PRACTICE & PROBLEM SOLVING

APPLY

27. Model with Mathematics The equation $r = 90 - 25\log(t + 1)$ is to model a student's retention r after taking a physics course where r represents a student's test score (as a percent), and t represents the number of months since taking the course.

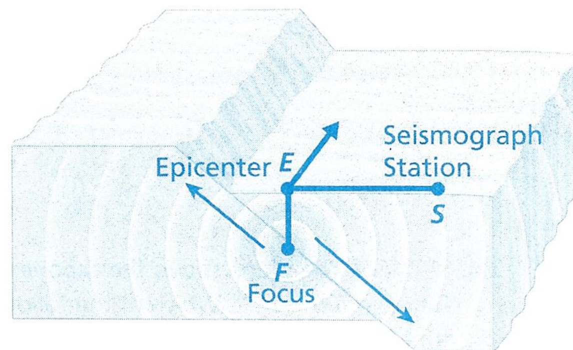
a. Make a table of values for ordered pairs that represent $r = 90 - 25\log(t + 1)$, rounding to the nearest tenth. Then sketch the graph of the function on a coordinate plane through those ordered pairs. (You may use a graphing calculator to check.)

b. Find the equation of the inverse. Interpret the meaning of this function.

28. Higher Order Thinking As shown by the diagram, an earthquake occurs below Earth's surface at point F (the focus). Point E, on the surface above the focus, is called the *epicenter*. A seismograph station at point S records the waves of energy generated by the earthquake. The surface wave magnitude M of the earthquake is given by this formula:

$$M = \log\left(\frac{A}{T}\right) + 1.66(\log D) + 3.3$$

In the formula, A is the amplitude of the ground motion in micrometers, T is the period in seconds, and D is the measure of $\overset{\frown}{ES}$ in degrees.



a. Find surface wave magnitude of an earthquake with $A = 700$ micrometers, $T = 2$ and $D = 100°$.

b. In the formula, $20° < D \leq 160°$. By how much can the size of arc ES affect the surface wave magnitude? Explain.

ASSESSMENT PRACTICE

29. The logarithmic function $g(x) = \ln x$ is transformed to $h(x) = \ln(x + 2) - 1$. Which of the following are true? Select **all** that apply.

Ⓐ $g(x)$ is translated 2 units upward.

Ⓑ $g(x)$ is translated 2 units to the right.

Ⓒ $g(x)$ is translated 2 units to the left.

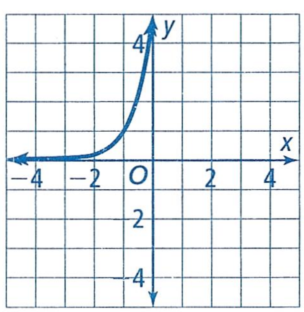
Ⓓ $g(x)$ is translated 1 unit downward.

Ⓔ $g(x)$ is translated 1 unit to the left.

Ⓕ The vertical asymptote shifts 2 units to the left.

Ⓖ The vertical asymptote shifts 2 units to the right.

30. SAT/ACT The graph shows the exponential function $f(x) = 5^{x+1}$. Which of the following functions represents its inverse, $f^{-1}(x)$?



Ⓐ $f^{-1}(x) = 1 + \log_5 x$

Ⓑ $f^{-1}(x) = \log_5 x - 1$

Ⓒ $f^{-1}(x) = \log_5 (x - 1)$

Ⓓ $f^{-1}(x) = \log_5 (x + 1)$

31. Performance Task The logarithmic function $M(d) = 5 \log d + 2$ is used to find the limiting magnitude of a telescope, where d represents the diameter of the lens of the telescope (mm) that is being used for the observation.

Part A Find the limiting magnitude of a telescope having a lens diameter of 40 mm.

Part B Find the equation of the inverse of this function.

Part C Interpret why astronomers may wish to use the inverse of this function. Justify your reasoning.

Part D Using the inverse function, find the diameter of the lens that has a limiting magnitude of 13.5. Check your answer with the table function of your graphing calculator.

Activity

6-5 Properties of Logarithms

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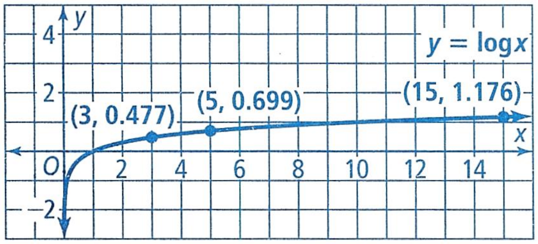
EXPLORE & REASON

Look at the graph of $y = \log x$ and the ordered pairs shown.



A. Complete the table shown.

x	3	5	15
$\log x$			

B. **Look for Relationships** What is the relationship between the numbers 3, 5, and 15? What is the relationship between the logarithms of 3, 5, and 15?

C. What is your prediction for the value of log 45? log 75? Explain.

HABITS OF MIND

Generalize Do you think that the relationships you found in the Explore & Reason activity would also hold for natural logarithms? Give an example.

Assess

EXAMPLE 1

Try It! Prove a Property of Logarithms

1. Prove the Quotient Property of Logarithms.

EXAMPLE 2

Try It! Expand Logarithmic Expressions

2. Use the properties of logarithms to expand each expression.

a. $\log_7\left(\frac{r^3t^4}{v}\right)$

b. $\ln\left(\frac{7}{225}\right)$

EXAMPLE 3

Try It! Write Expressions as Single Logarithms

3. Write each expression written as a single logarithm.

a. $5\log_2 c - 7\log_2 n$

b. $2\ln 7 + \ln 2$

HABITS OF MIND

Make Sense and Persevere Using the fact that $\log 2 \approx 0.3010$ and $\log 3 \approx 0.4771$, what is $\log 18$? Show you know.

EXAMPLE 4

Try It! Apply Properties of Logarithms

4. What is the concentration of hydrogen ions in a liter of orange juice?

HABITS OF MIND

Generalize What types of numbers have logarithms that are negative? Explain.

EXAMPLE 5

Try It! Evaluate Logarithmic Expressions by Changing the Base

5. Estimate the value of each logarithm. Then use a calculator to find the value of each logarithm to the nearest thousandth.

a. $\log_2 7$

b. $\log_5 3$

EXAMPLE 6

Try It! Use the Change of Base Formula

6. What is the solution to the equation $3^x = 15$? Express the solution as a logarithm and then evaluate. Round to the nearest thousandth.

HABITS OF MIND

Use Appropriate Tools Why is the Change of Base Formula useful when evaluating a logarithm with a calculator?

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How are the properties of logarithms used to simplify expressions and solve logarithmic equations?

2. **Vocabulary** While it is not necessary to change to base 10 when applying the Change of Base Formula, why is it common to do so?

3. **Error Analysis** Amanda claimed the expanded form of the expression $\log_4(c^2d^5)$ is $5\log_4 c + 5\log_4 d$. Explain the error Amanda made.

Do You KNOW HOW?

4. Use the properties of logarithms to expand the expression $\log_6\left(\frac{49}{5}\right)$.

5. Use the properties of logarithms to write the expression $5 \ln s + 6 \ln t$ as a single logarithm.

6. Use the formula $\text{pH} = \log\frac{1}{[H^+]}$ to write an expression for the concentration of hydrogen ions, $[H^+]$, in a container of baking soda with a pH of 8.9.

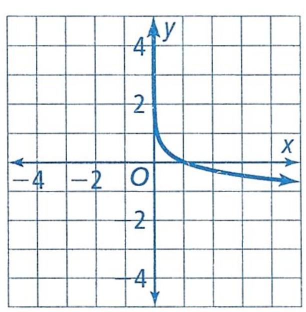
PRACTICE & PROBLEM SOLVING

UNDERSTAND

7. **Use Structure** Without applying the Change of Base Formula, explain how to use $\log_3 2 \approx 0.631$ and $\log_3 5 \approx 1.465$ to approximate $\log_3\left(\frac{2}{5}\right)$.

8. **Communicate Precisely** Explain what is meant by *expanding a logarithmic expression*. How are the processes of *expanding logarithmic expressions* and *writing logarithmic expressions as a single logarithm* related?

9. **Higher Order Thinking** The graph of $y = \log\left(\frac{1}{x}\right)$ and $y = -\log x$ are shown. Notice the graph is the same for both equations. Use properties of logarithms to explain why the graphs are the same.



10. **Communicate Precisely** Emma used the Change of Base Formula to solve the equation $6^x = 72$ and found that $x = 2.387$. How can Emma check her solution?

11. **Error Analysis** Describe and correct the error a student made in writing the logarithmic expression in terms of a single logarithm.

$$\log_3 2 + \frac{1}{2}\log_3 y = \log_3 2y^2 \quad ✗$$

12. **Error Analysis** A student wants to approximate $\log_2 9$ with her calculator. She enters the equivalent expression $\frac{\ln 2}{\ln 9}$, but the decimal value is not close to her estimate of 3. What happened?

$$\log_2 9 = \frac{\ln 2}{\ln 9} \quad ✗$$

PRACTICE & PROBLEM SOLVING

PRACTICE

13. Use the properties of exponents to prove the Power Property of Logarithms. SEE EXAMPLE 1

Use the properties of logarithms to expand each expression. SEE EXAMPLE 2

14. $\log_5\left(\frac{2}{3}\right)$

15. $\log_6(2m^5n^3)$

16. $\ln 2x^5$

17. $\log_2\left(\frac{x}{5y}\right)$

Use the properties of logarithms to write each expression as a single logarithm. SEE EXAMPLE 3

18. $9\ln x - 6\ln y$

19. $\log_5 6 + \frac{1}{2}\log_5 y$

20. $2\log 10 + 4\log(3x)$

21. $\frac{1}{3}\ln 27 - 3\ln(2y)$

22. $8\log_3 2 + 5\log_3 c + 7\log_3 d$

23. Use properties of logarithms to show that $\text{pH} = \log\frac{1}{[H^+]}$ can be written as $\text{pH} = -\log[H^+]$.
SEE EXAMPLE 4

Use the Change of Base Formula to evaluate each logarithm. Round to the nearest thousandth.
SEE EXAMPLE 5

24. $\log_4 9$

25. $\log_6 5$

26. $\ln 3$

27. $\log_2 7$

28. $\log_9 12$

29. $\ln 23$

Use the Change of Base Formula to solve each equation for *x*. Give an exact solution as a logarithm and an approximate solution rounded to the nearest thousandth. SEE EXAMPLE 6

30. $3^x = 4$

31. $5^x = 11$

32. $8^x = 10$

33. $2^x = 30$

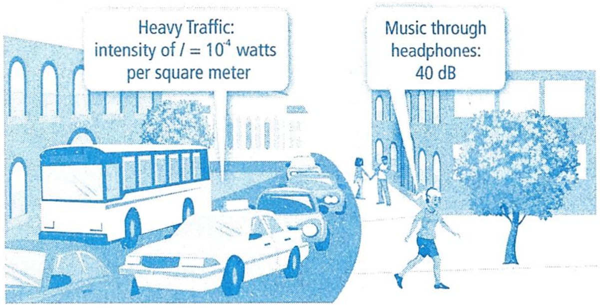
34. $7^x = 100$

35. $4^x = 55$

PRACTICE & PROBLEM SOLVING

APPLY

36. Make Sense and Persevere The loudness of sound is measured in decibels. For a sound with intensity I (in watts per square meter), its loudness $L(I)$ (in decibels) is modeled by the function $L(I) = 10\log\frac{I}{I_0}$, where I_0 represents the intensity of a barely audible sound (approximately 10^{-12} watts per square meter).



a. Find the decibel level of the sound made by the heavy traffic.

b. Find the intensity of the sound that is made by music playing at 40 decibels.

c. How many times as great is the intensity of the traffic than the intensity of the music?

37. Model with Mathematics Miguel collected data on the attendance at an amusement park and the daily high temperature. He found that the model $A = 2\log t + \log 5$ approximated the attendance A, in thousands of people, at the amusement park, when the daily high temperature is t degrees Fahrenheit.

a. Use properties of logarithms to simplify Miguel's formula.

b. The daily high temperatures for the week are below.

Mon	Tue	Wed	Thu	Fri
87° 66°	73° 67°	65° 61°	72° 57°	80° 59°

Daily temperatures show highs and lows in °F.

What is the expected attendance on Wednesday? Round to the nearest person.

ASSESSMENT PRACTICE

38. Match each expression with an equivalent expression.

I. $\log_4 20$	Ⓐ $\log_2 20 - \log_2 4$
II. $2\log_2 5$	Ⓑ $\log_4 2 + \log_4 10$
III. $\log_2 5$	Ⓒ $\frac{\log 25}{\log 2}$
VI. $4\log_4 2$	Ⓓ $\log_2 4$

39. **SAT/ACT** Use the properties of logarithms to write the following expression in terms of a single logarithm.

$$2(\log_3 20 - \log_3 4) + 0.5 \log_3 4$$

Ⓐ $\log_3 4$
Ⓑ $\log_3 5$
Ⓒ $\log_3 25$
Ⓓ $\log_3 50$

40. **Performance Task** The magnitude, or intensity, of an earthquake is measured on the Richter scale. For an earthquake where the amplitude of its seismographic trace is A, its magnitude is modeled by the function:

$$R(A) = \log \frac{A}{A_0},$$

where A_0 represents the amplitude of the smallest detectable earthquake.

Part A An earthquake occurs with an amplitude 200 times greater than the amplitude of the smallest detectable earthquake, A_0. What is the magnitude of this earthquake on the Richter scale?

Part B Approximately how many times as great is the amplitude of an earthquake measuring 6.8 on the Richter scale than the amplitude of an earthquake measuring 5.9 on the Richter scale?

Part C Suppose the intensity of one earthquake is 150 times as great as that of another. How much greater is the magnitude of the more intense earthquake than the less intense earthquake?

MODEL & DISCUSS

A store introduces two new models of fitness trackers to its product line. A glance at the data is enough to see that sales of both types of fitness trackers are increasing. Unfortunately, the store has limited space for the merchandise. The manager decides that the store will sell both models until sales of TrackSmart exceed those of FitTracker.

	FitTracker *Number Sold*	**TrackSmart** *Number Sold*
Week 4	228	130
Week 3	112	44
Week 2	54	17
Week 1	28	5

A. **Model With Mathematics** Find an equation of an exponential that models the sales for each fitness tracker. Describe your method.

B. Based on the equations that you wrote, determine when the store will stop selling FitTracker.

HABITS OF MIND

Look for Relationships How do you know that the sales data is modeled by an exponential function?

EXAMPLE 1

Try It! Solve Exponential Equations Using a Common Base

1. Solve each equation using a common base.

   a. $25^{3x} = 125^{x+2}$

   b. $0.001 = 10^{6x}$

EXAMPLE 2

Try It! Rewrite Exponential Equations Using Logarithms

2. Rewrite the equation $5^x = 12$ using logarithms.

HABITS OF MIND

Communicate Precisely In order to set the exponents of two exponential expressions equal to each other, what must be true about the exponential expressions?

Notes

Assess

EXAMPLE 3 Try It! Solve Exponential Equations Using Logarithms

3. What is the solution to $2^{3x} = 7^{x+1}$?

EXAMPLE 4 Try It! Use an Exponential Model

4. About how many minutes does it take the fire to spread to cover 100 acres?

HABITS OF MIND

Use Structure Why is it useful to use logarithms to solve an exponential equation?

EXAMPLE 5 Try It! Solve Logarithmic Equations

5. Solve each equation.

a. $\log_5 (x^2 - 45) = \log_5 (4x)$

b. $\ln (-4x - 1) = \ln (4x^2)$

EXAMPLE 6 Try It! Solve Logarithmic and Exponential Equations by Graphing

6. Solve each equation by graphing. Round to the nearest thousandth.

a. $3(2)^{x+2} - 1 = 3 - x$

b. $\ln (3x - 1) = x - 5$

HABITS OF MIND

Generalize Summarize the procedure for solving a logarithmic equation.

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do properties of exponents and logarithms help you solve equations?

2. **Vocabulary** Jordan claims that $x^2 + 3 = 12$ is an exponential equation. Is Jordan correct? Explain your thinking.

3. **Communicate Precisely** How can properties of logarithms help to solve an equation such as $\log_6 (8x - 2)^3 = 12$?

Do You KNOW HOW?

Solve. Round to the nearest hundredth, if necessary. List any extraneous solutions.

4. $16^{3x} = 256^{x+1}$

5. $6^{x+2} = 4^x$

6. $\log_5 (x^2 - 44) = \log_5 (7x)$

7. $\log_2 (3x - 2) = 4$

8. $4^{2x} = 9^{x-1}$

9. A rabbit farm had 200 rabbits in 2015. The number of rabbits increases by 30% every year. How many rabbits are on the farm in 2031?

PRACTICE & PROBLEM SOLVING

UNDERSTAND

10. **Use Structure** Would you use the natural log or the common log when solving the equation $10^{x+2} = 78$? Is it possible to use either the natural log or common log? Explain.

11. **Make Sense and Persevere** Explain why logarithms are necessary to solve the equation $3^{x+2} = 8$, but are not necessary to solve the equation $3^{x+2} = 27^{4x}$.

12. **Reason** Tristen solved the equation $\log_3 (x + 1) - \log_3 (x - 6) = \log_3 (2x + 2)$. Justify each step of solving the equation in Tristen's work. Are both numbers solutions to the equation? Explain.

$$\log_3(x + 1) - \log_3(x - 6) = \log_3(2x + 2)$$
$$\log_3(x + 1) = \log_3(2x + 2) + \log_3(x - 6)$$
$$\log_3(x + 1) = \log_3(2x + 2)(x - 6)$$
$$(x + 1) = (2x + 2)(x - 6)$$
$$x + 1 = 2x^2 - 10x - 12$$
$$0 = 2x^2 - 11x - 13$$
$$x = 6.5 \text{ or } x = -1$$

13. **Error Analysis** The number of milligrams of medicine in a person's system after t hours is given by the function $A = 20e^{-0.40t}$. Thomas sets $A = 0$ to find the number of hours it takes for all of the medicine to be removed from a person's system. What mistake did Thomas make? Explain.

14. **Mathematical Connections** Explain the importance of the Power Property of Logarithms when solving exponential equations.

15. **Error Analysis** Find the student error in the solution of the logarithmic equation.

$$\log (x + 3) + \log x = 1$$
$$\log x (x + 3) = 1$$
$$x(x + 3) = 10^1$$
$$x^2 + 3x - 10 = 0$$
$$(x - 2)(x + 5) = 0$$
$$x = 2, -5$$

X

PRACTICE & PROBLEM SOLVING

PRACTICE

Find all solutions of the equation. Round answers to the nearest ten-thousandth. SEE EXAMPLE 1

16. $3^{2-3x} = 3^{5x-6}$

17. $7^{3x} = 54$

18. $25^{x^2} = 125^{x+3}$

19. $4^{3x-1} = \left(\frac{1}{2}\right)^{x+5}$

20. $4^{2x+1} = 4^{3x-5}$

21. $6^{x-2} = 216$

Find all solutions of the equation. Round answers to the nearest ten-thousandth. SEE EXAMPLES 2 AND 3

22. $2^{3x-2} = 5$

23. $4 + 5^{6-x} = 15$

24. $6^{3x+1} = 9^x$

25. $-3 = \left(\frac{1}{2}\right)^x - 12$

26. $3^{2x-3} = 4^x$

27. $4^{x+2} = 8^{x-1}$

28. Dale has $1,000 to invest. He has a goal to have $2,500 in this investment in 10 years. At what annual rate compounded continuously will Dale reach his goal? Round to the nearest hundredth. SEE EXAMPLE 4

Find all solutions of the equation. Round answers to the nearest thousandth. SEE EXAMPLE 5

29. $\log_2 (4x + 5) = \log_2 x^2$

30. $2\ln (3x - 2) = \ln (5x + 6)$

31. $\log_4 (x^2 - 2x) = \log_4 (3x + 8)$

32. $\ln (5x - 2) = \ln (x - 1)$

33. $\ln (2x^2 + 5x) = \ln (2x + 7)$

34. $2\log (x + 1) = \log (x + 1)$

35. $\log_2 x + \log_2 (x - 3) = 2$

36. $\log_2 (3x - 2) = \log_2 (x - 1) + 4$

37. $\log_6 (x^2 - 2x) = \log_6 (2x - 3) + \log_6 (x + 1)$

Solve by graphing. Round answers to the nearest thousandth. SEE EXAMPLE 6

38. $\log(5x - 3)^2 = x - 4$

39. $\ln(2x) = 3x - 5$

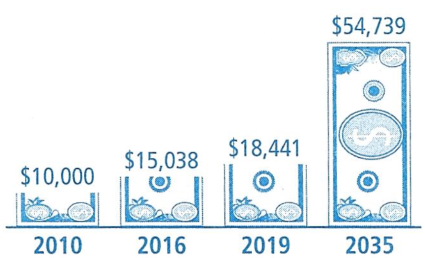
40. $\log(4x) = x + \log x$

PRACTICE & PROBLEM SOLVING

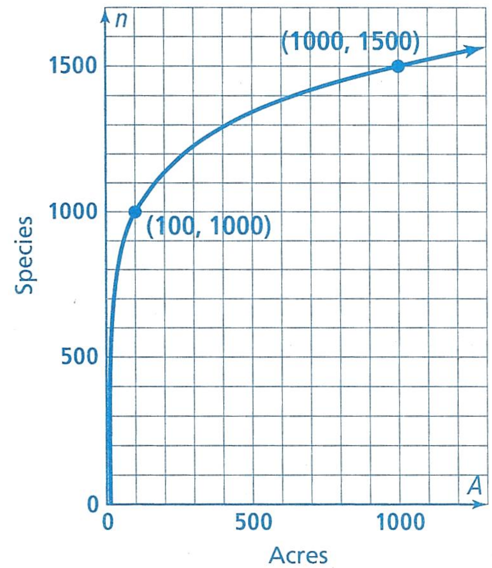
APPLY

41. Model With Mathematics The population of a city is modeled by the function $P = 250{,}000e^{0.013t}$, where t is the number of years since 2000. In what year, to the nearest year, will the population reach 450,000?

42. Use Structure Felix invested \$10,000 into a retirement account in 2010. He then projected the amount of money that would be in the account for several years assuming that interest would compound continuously at an annual rate. Later, when he looked back the data, he could not recall the annual rate that he used for the projections. Use the data below to determine the annual rate.



43. Higher Order Thinking A biologist is using the logarithmic model $n = k\log(A)$ to determine the number of a species n, that can live on a land mass of area A. The constant k varies according to the species.



a. Use the graph to determine the constant k for the species that the scientist is studying.

b. Determine the land mass in acres that is needed to support 3,000 of the species.

ASSESSMENT PRACTICE

44. Which of the following have the same solution? Select all that apply.

Ⓐ $\log_8 (x^2 - 15) = \log_8 (2x)$

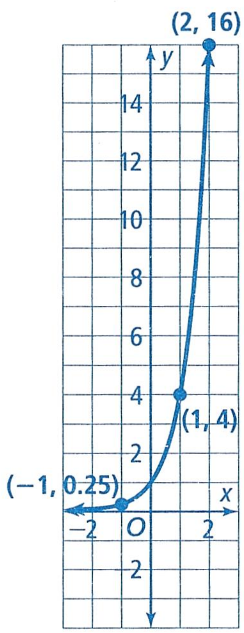
Ⓑ $\ln (12x + 2) = \ln (2x - 3)$

Ⓒ $\log_2 x + \log_2 (x + 4) = 5$

Ⓓ $\log_3 (15x + 6)^2 = 8$

Ⓔ $\log_4 (3x - 5) = 2$

45. SAT/ACT The graph shows the function $y = 4^x$. Determine when the function shown in the graph is greater than the function $y = 2^{3x-1}$.



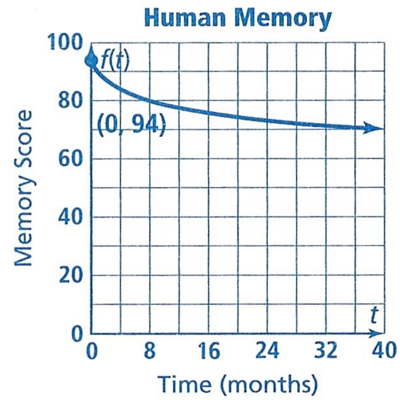
Ⓐ $x > 1$

Ⓑ $x < 1$

Ⓒ $x > -1$

Ⓓ $x < -1$

46. Performance Task A professor conducted an experiment to find the relationship between time and memory. The professor determined the model $f(t) = t_0 - 15 \log (t + 1.1)$ gives the memory score after t months when a student had an initial memory score of t_0.



Part A Write a model for a student with the given initial memory score.

Part B After about how many years will the student have a memory score of 65?

EXPLORE & REASON

A store offered customers two plans for getting bonus points:

A. What expression represents the number of points received each day for Plan A?

B. What expression represents the number of points received each day for Plan B?

C. Reason On the 7th day, which plan would offer the most bonus points? Explain.

HABITS OF MIND

Reason Does Plan B always offer more points than Plan A? Explain.

EXAMPLE 1

Try It! Identify Geometric Sequences

1. Is the sequence a geometric sequence? If so, write a recursive definition for the sequence.

   a. 1.22, 1.45, 1.68, 1.91, ...

   b. −1.5, 0.75, −0.375, 0.1875, ...

EXAMPLE 2

Try It! Translate Between Recursive and Explicit Definitions

2. a. Given the recursive definition $a_n = \begin{cases} 12, & n = 1 \\ \frac{1}{3}a_{n-1}, & n > 1 \end{cases}$;
   what is the explicit definition for the sequence?

   b. Given the explicit definition $a_n = 6(1.2)^{n-1}$;
   what is the recursive definition?

EXAMPLE 3

Try It! Solve Problems with Geometric Sequences

3. A geometric sequence can be used to describe the growth of bacteria in an experiment. On the first day of the experiment there were 9 bacteria in a Petri dish. On the 10th day, there are 3^{20} bacteria in the dish. How many bacteria were in the dish on the 7th day of the experiment?

HABITS OF MIND

Use Appropriate Tools How can you use the recursive definition for a geometric sequence to find the 19th term?

Notes

Assess

EXAMPLE 4 **Try It!** **Formula for the Sum of a Finite Geometric Series**

4. a. Write the expanded form of the series $\sum_{n=1}^{5} \frac{1}{2}(3)^{n-1}$. What is the sum?

b. Write the series $-2 + \left(\frac{-2}{3}\right) + \ldots + \left(\frac{-2}{243}\right)$ using sigma notation. What is the sum?

EXAMPLE 5 **Try It!** **Find the Number of Terms in a Finite Geometric Series**

5. a. How many terms are in the geometric series $3 + 6 + 12 + \ldots + 768$?

b. The sum of a geometric series is 155. The first term of the series is 5, and its common ratio is 2. How many terms are in the series?

EXAMPLE 6 **Try It!** **Use a Finite Geometric Series**

6. What is the monthly payment for a $40,000 loan for 4 years with an annual interest rate of 4.8%?

HABITS OF MIND

Make Sense and Persevere Why is using a formula easier than calculating and adding all 10 terms?

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you represent and use geometric sequences and series?

2. **Error Analysis** Denzel claims the sequence 0, 7, 49, 343, … is a geometric sequence and the next number is 2,401. What error did he make?

3. **Vocabulary** Describe the similarities and differences between a common difference and a common ratio.

4. **Use Structure** What happens to the terms of a sequence if a_1 is positive and $r > 1$? What happens if $0 < r < 1$? Explain.

Do You KNOW HOW?

Find the common ratio and the next three terms of each geometric sequence.

5. 2, −4, 8, −16, …

6. −64, −16, −4, −1, …

7. 0.8, 2.4, 7.2, 21.6, …

8. 2, −10, 50, −250, …

9. 100, 50, 25, 12.5, …

10. In a video game, players earn 10 points for finishing the first level and twice as many points for each additional level. How many points does a player earn for finishing the fifth level? How many points will the player have earned in the game up to that point?

PRACTICE & PROBLEM SOLVING

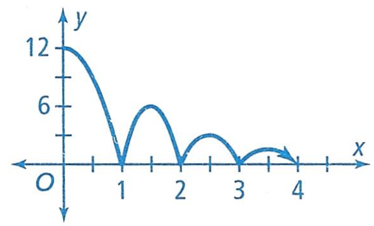
UNDERSTAND

11. **Reason** True or False: If the first two terms of a geometric sequence are positive, then the third term is positive. Explain your reasoning.

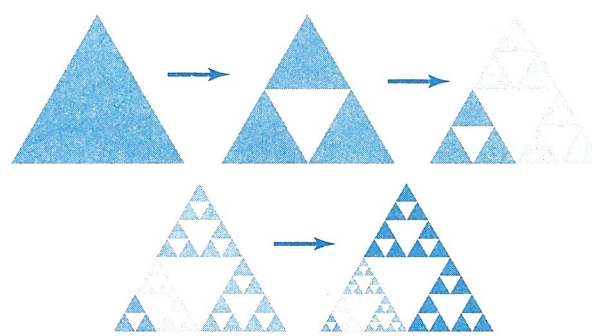
12. **Error Analysis** The first term of a geometric sequence is 4 and grows exponentially by a factor of 3. Murphy writes out the terms and says that the sum of the 4th and 5th terms is 1,296. Explain Murphy's error and correct it.

13. **Construct Arguments** Write a geometric sequence with at least four terms and describe it using both an explicit and recursive definitions. How can you confirm that your sequence is geometric?

14. **Higher Order Thinking** Adam drops a ball from a height of 12 feet. Each bounce is 50% as high as the previous bounce. What is the total vertical distance the ball has traveled when it hits the ground for the 4th time?



15. **Model with Mathematics** The Sierpinski Triangle is a fractal made by cutting an equilateral triangle into four congruent pieces and removing the center piece, leaving three smaller triangles. The process is repeated on each triangle, creating more triangles that are even smaller. Continuing this pattern, how many triangles would there be after the tenth step in the process?



PRACTICE & PROBLEM SOLVING

PRACTICE

Is the sequence geometric? If so, write a recursive definition for the sequence. SEE EXAMPLE 1

16. 1, −3, 9, −27, ...

17. 3, −15, 75, −375, ...

18. 4, 5, 6, 7, ...

19. 24, 8, $\frac{8}{3}$, $\frac{8}{9}$, ...

20. 2, 4, 6, 8, ...

21. 10, 40, 160, 640, ...

Translate between the recursive and explicit definitions for each sequence. SEE EXAMPLE 2

22. $a_n = 1{,}024\left(\frac{1}{2}\right)^{n-1}$

23. $a_n = \begin{cases} 2, & n = 1 \\ -2a^{n-1}, & n > 1 \end{cases}$

24. $a_n = 35(2)^{n-1}$

25. $a_n = -6(-3)^{n-1}$

26. $a_n = \begin{cases} 1, & n = 1 \\ \frac{2}{3}a_{n-1}, & n > 1 \end{cases}$

27. In an experiment, the number of bacteria present each day form a geometric sequence. On the first day, there were 100 bacteria. On the eighth day, there were 12,800 bacteria. How many bacteria were there on the fourth day? SEE EXAMPLE 3

Write the expansion of each series. What is the sum? SEE EXAMPLE 4

28. $\sum_{n=1}^{6} 4(2)^{n-1}$

29. $\sum_{n=1}^{20} 6(2)^{n-1}$

30. $\sum_{n=1}^{7} -4(3)^{n-1}$

31. $\sum_{n=1}^{12} (-4)^{n-1}$

Write each series using sigma notation. Find the sum. SEE EXAMPLE 4

32. 8 + 16 + 32 + ... + 1,024

33. −7 − 42 − 252 − ... − 54,432

34. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \ldots + \frac{1}{80}$

35. 4 − 12 + 36 − ... + 2,916

36. The sum of a geometric series is 31.75. The first term of the series is 16, and its common ratio is 0.5. How many terms are in the series? SEE EXAMPLE 5

37. What is the monthly payment for a $12,000 loan for 7 years with an annual interest rate of 2.7%? SEE EXAMPLE 6

PRACTICE & PROBLEM SOLVING

APPLY

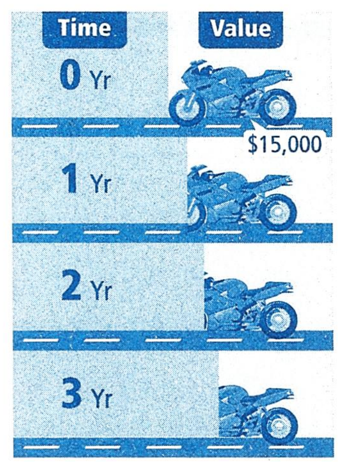
38. **Model With Mathematics** Kelley opens a bank account to save for a down payment on a car. Her initial deposit is $250, and she plans to deposit 10% more each month. Kelley's goal is to have $2,000 in the account after six months. Will she meet her goal?

39. **Make Sense and Persevere** Henry just started his own cleaning business. He is using word-of-mouth from his current clients to promote his business. He currently has seven clients.

    a. Five of his clients really like Henry's work and each told two friends the following month. This group each told two friends the following month, and so on for a total of five months. Assuming no one heard twice, how many people have had or heard of a positive experience with Henry's cleaning business?

    b. The two unhappy clients each told five people the following month. This group each told five people, and so on, for five months. Assuming no one heard twice, how many people have had or heard of a negative experience with Henry's cleaning business?

40. **Model with Mathematics** Ricardo bought a motorcycle for $15,000. The value depreciates 15% at the start of every year. What is the value of the motorcycle after three years?



ASSESSMENT PRACTICE

41. The first term of a geometric series is −1, and the common ratio is −2. Fill in the number to complete the sentence.

If the sum of the series is −43, there are ___?___ terms in the series.

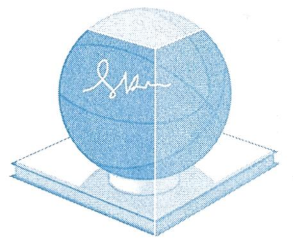
42. SAT/ACT What is the value of the 11th term in the following geometric sequence?

$$\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots$$

Ⓐ 3^4
Ⓑ 3^5
Ⓒ 3^6
Ⓓ 3^7
Ⓔ 3^8

43. Performance Task An avid collector wants to purchase a signed basketball from a particular playoff game. He plans to put away 4% more money each year, in a safe at his home, to save up for the basketball. In the sixth year, he puts \$580 in the safe and realizes that he has exactly enough money to purchase the basketball.

Price = Year 5 savings + \$580.00



Part A How much money did the collector put into the safe the first year?

Part B To the nearest dollar, how much did the collector pay for the signed playoff basketball?