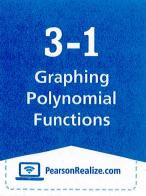## EXPLORE & REASON

Consider functions of the form $f(x) = x^n$, where n is a positive integer.

A. Graph $f(x) = x^n$ for $n = 1$, 3, and 5. Look at the graphs in Quadrant I. As the exponent increases, what is happening to the graphs? Which quadrants do the graphs pass through?

B. Look for Relationships Now graph $f(x) = x^n$ for $n = 2$, 4, and 6. What happens to these graphs in Quadrant I as the exponent increases? Which quadrants do the graphs pass through?

C. Write two equations in the form $f(x) = x^n$ with graphs that you predict are in Quadrants I and II. Write two equations with graphs that you predict are in Quadrants I and III. Use graphing technology to test your predictions.

HABITS OF MIND

Construct Arguments Compare and contrast the end behavior of the graphs of $f(x) = x^n$ when $n = 1$, 3, & 5 with the graphs of $f(x) = x^n$ when $n = 2$, 4, & 10. Write a general statement that compares the end behavior of the graphs when the exponents are odd to the end behavior when the exponents are even.

EXAMPLE 1 ☑ **Try It!** Classify Polynomials

1. What is each polynomial in standard form. What are the leading coefficient, the degree, and the number of terms of each?

   a. $2x - 3x^4 + 6 - 5x^3$ b. $x^5 + 2x^6 - 3x^4 - 8x + 4x^3$

EXAMPLE 2 ☑ **Try It!** Interpret Leading Coefficients and Degrees

2. Use the leading coefficient and degree of the polynomial function to determine the end behavior of each graph.
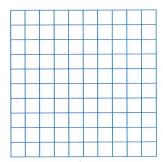
   a. $f(x) = 2x^6 - 5x^5 + 6x^4 - x^3 + 4x^2 - x + 1$

   b. $g(x) = -5x^3 + 8x + 4$

HABITS OF MIND

Communicate Precisely How does the leading coefficient help determine the end behavior of an even function?
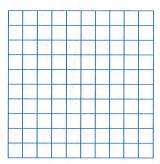
EXAMPLE 3 ☑ **Try It!** Graph a Polynomial Function

3. Consider the polynomial function $f(x) = x^5 + 18x^2 + 10x + 1$.

   a. Make a table of values to identify key features and sketch a graph of the function.

   b. Find the average rate of change over the interval [0, 2].

EXAMPLE 4 ☑ **Try It!** **Sketch a Graph from a Verbal Description**

4. Use the information below to sketch a graph of the polynomial function $y = f(x)$.

- $f(x)$ is positive on the intervals $(-2, -1)$ and $(1, 2)$.
- $f(x)$ is negative on the intervals $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$.
- $f(x)$ is increasing on the intervals $(-\infty, -1.5)$ and $(0, 1.5)$.
- $f(x)$ is decreasing on the intervals $(-1.5, 0)$ and $(1.5, \infty)$.

HABITS OF MIND

Generalize What can you tell about the graph of a function if its equation has an odd degree and a negative leading coefficient?

EXAMPLE 5 ☑ **Try It!** **Interpret a Polynomial Model**

5. Danielle is engineering a new brand of shoes. For x shoes sold, in thousands, a profit of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$ dollars, in ten thousands, will be earned.

a. How much will be earned in profit for selling 1,000 shoes?

b. What do the x- and y-intercepts of the graph mean in this context? Do those values make sense?

HABITS OF MIND

Use Appropriate Tools Estimate the turning point of the graph of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$. What does this point represent in the context of Try It! 5?

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do the key features of a polynomial function help you sketch its graph?
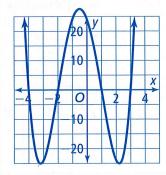
2. Error Analysis Allie said the degree of the polynomial function $f(x) = x^5 + 2x^4 + 3x^3 - 2x^6 - 9x^2 - 6x + 4$ is 5. Explain and correct Allie's error.

3. Vocabulary Explain how to determine the **leading coefficient** of a polynomial function.

4. Look for Relationships What is the relationship between the degree and leading coefficient of a polynomial function and the end behavior of the polynomial?

Do You KNOW HOW?

The graph shows the function $f(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$. Find the following.

5. number of terms

6. degree

7. leading coefficient

8. end behavior

9. turning point(s)

10. x-intercept(s)

11. relative minimum(s)

12. relative maximum(s)

PRACTICE & PROBLEM SOLVING

UNDERSTAND

13. Make Sense and Persevere The table shows some values of a polynomial function. Deshawn says there are turning points between the x-values −3 and −2 and between 0 and 1. He also says there is a relative minimum between the x-values −3 and −2, and a relative maximum between 0 and 1. Sketch a graph that shows how Deshawn could be correct and another graph that shows how Deshawn could be incorrect.

x	−5	−4	−3	−2	−1	0	1	2
f(x)	−1004	129	220	85	12	1	4	165

14. Higher Order Thinking Use the information below about a polynomial function in standard form to write a possible polynomial function. Explain how you determined your function and graph it to verify that it satisfies the criteria.

- 6 terms
- y-intercept at 1
- end behavior: As $x \to -\infty$, $y \to +\infty$.
  As $x \to +\infty$, $y \to -\infty$.

15. Reason An analyst for a new company used the first three years of revenue data to project future revenue for the company. The analyst predicts the function $f(x) = -2x^5 + 6x^4 - x^3 + 5x^2 + 6x + 50$ will give the revenue after x years. Should the CEO expect the company to be successful? Explain.

16. Look for Relationships Sketch a graph of each of the functions described below.

- a cubic function with one x-intercept
- a cubic function with 2 x-intercepts
- a cubic function with 3 x-intercepts

17. Make Sense and Persevere Compare the rate of change for the function $f(x) = x^3 - 2x^2 + x + 1$ over the intervals [0, 2] and [2, 4].

PRACTICE & PROBLEM SOLVING

PRACTICE

Write each polynomial function in standard form. For each function, find the degree, number of terms, and leading coefficient. SEE EXAMPLE 1

18. $f(x) = -3x^3 + 2x^5 + x + 8x^3 - 6 + x^4 - 3x^2$

19. $f(x) = 8x^2 + 10x^7 - 7x^3 - x^4$

20. $f(x) = -x^3 + 9x + 12 - x^4 + 5x^2$

Use the leading coefficient and degree of the polynomial function to determine the end behavior of the graph. SEE EXAMPLE 2

21. $f(x) = -x^5 + 2x^4 + 3x^3 + 2x^2 - 8x + 9$

22. $f(x) = 7x^4 - 4x^3 + 7x^2 + 10x - 15$

23. $f(x) = -x^6 + 7x^5 - x^4 + 2x^3 + 9x^2 - 8x - 2$

Use a table of values to estimate the intercepts and turning points of the function. Then graph the function. SEE EXAMPLE 3
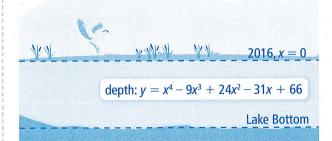
24. $f(x) = x^3 + 2x^2 - 5x - 6$

25. $f(x) = x^4 - x^3 - 21x^2 + x + 20$

26. Use the information below to sketch a graph of the polynomial function $y = f(x)$.
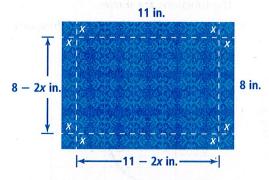   SEE EXAMPLE 4

    • $f(x)$ is positive on the intervals $(-\infty, -3)$, $(-2, 0)$, and $(2, 3)$.

    • $f(x)$ is negative on the intervals $(-3, -2)$, $(0, 2)$, and $(3, \infty)$.

    • $f(x)$ is increasing on the interval $(-2.67, -1)$ and $(1, 2.5)$.

    • $f(x)$ is decreasing on the intervals $(-\infty, -2.67)$, $(-1, 1)$, and $(2.5, \infty)$.

27. The equation shown models the average depth y, in feet, of a lake, x years after 2016, where $0 < x < 6$. Use technology to graph the function. In what year does this model predict a relative minimum value for the depth? SEE EXAMPLE 5

    2016, $x = 0$

    depth: $y = x^4 - 9x^3 + 24x^2 - 31x + 66$

    Lake Bottom

PRACTICE & PROBLEM SOLVING

APPLY

28. Reason Allie has a piece of construction paper that she wants to use to make an open rectangular prism. She will cut a square with side length x from each corner of the paper, so the length and width is decreased by $2x$ as shown in the diagram.

11 in.

8 − 2x in.

8 in.

11 − 2x in.

a. Write a function that models the volume of the rectangular prism.

b. Graph the function and identify a reasonable domain.

c. What do the x-intercepts of the graph mean in this context?

d. If Allie wants to maximize the volume of the box, what is the side length of the squares that should be cut from each corner of the piece of construction paper? Explain.

29. Make Sense and Persevere Alberto is designing a container in the shape of a rectangular prism to ship electronic devices. The length of the container is 10 inches longer than the height. The sum of the length, width, and height is 25 inches. The volume of the container, in terms of height x, is shown. Use a graphing calculator to graph the function. What do the x-intercepts of the graph mean in this context? What dimensions of the container will maximize the volume?
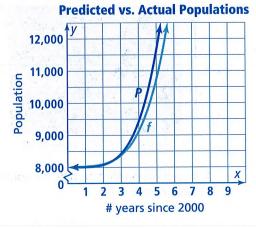
$$V = f(x) = -2x^3 - 5x^2 + 150x$$

x

$x + 10$

ASSESSMENT PRACTICE

30. Complete the table to give the leading coefficient of each polynomial function.

Polynomial Function	Leading coefficient
$f(x) = 3x^3 + 2x^2 + 9x - 6$	
$f(x) = -3x^4 + 8x^2 - 2x + 7x^5$	
$f(x) = -3x^5 + 7x^3 + 6x^2 - 2$	
$f(x) = 3x^2 - 12x^4 - 3x^6 - 3x^3$	
$f(x) = 6x^3 + 9x^2 - 5x - 3$	

31. SAT/ACT What is the maximum number of terms a fourth-degree polynomial function in standard form can have?

Ⓐ 1

Ⓑ 2

Ⓒ 3

Ⓓ 4

Ⓔ 5

32. Performance Task In the year 2000, a demographer predicted the estimated population of a city, which can be modeled by the function $f(x) = 5x^4 - 4x^3 + 25x + 8,000$. Several years later, a statistician, using data from the U.S. Census Bureau, modeled the actual population with the function $P(x) = 7x^4 - 6x^3 + 5x + 8,000$. The graphs of the functions are shown.

Predicted vs. Actual Populations

Part A What is the y-intercept of each function, and what does it represent?

Part B Identify the end behaviors of f and P.

Part C Compare the average rates of change of f and P from 2003 to 2005.

EXPLORE & REASON

Let S be the set of expressions that can be written as $ax + b$, where a and b are real numbers.

A. Describe the Associative Property, the Commutative Property, and the Distributive Property. Then, explain the role of each in simplifying the sum $(3x + 2) + (7x - 4)$ and identify the leading coefficient and the constant term in the result.

B. Is the sum you found in part A a member of S? Explain.

C. Construct Arguments Is the product of two expressions in S also a member of S? Explain why or produce a counterexample.

HABITS OF MIND

Construct Arguments Is the quotient of two expressions in S also a member of S? Explain why or produce a counterexample.

EXAMPLE 1 ☑ **Try It!** **Add and Subtract Polynomials**

1. Add or subtract the polynomials.

   a. $(4a^4 - 6a^3 - 3a^2 + a + 1) + (5a^3 + 7a^2 + 2a - 2)$

   b. $(2a^2b^2 + 3ab^2 - 5a^2b) - (3a^2b^2 - 9a^2b + 7ab^2)$

HABITS OF MIND

Generalize When can you combine two terms using addition or subtraction?

EXAMPLE 2 ☑ **Try It!** **Multiply Polynomials**

2. Multiply the polynomials.

   a. $(6n^2 - 7)(n^2 + n + 3)$

   b. $(mn + 1)(m^2n - 1)(mn^2 + 2)$

EXAMPLE 3 ☑ **Try It!** **Understand Closure**

3. Is the set of monomials closed under multiplication? Explain.

HABITS OF MIND

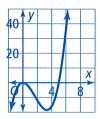**Construct Arguments** Is the set of polynomials closed under multiplication? Explain.

EXAMPLE 4 ☑ **Try It!** **Write a Polynomial Function**

4. The cost of Carolina's materials changes so that her new cost function is $c(x) = 4x + 42$. Find the new profit function. Then find the quantity that maximizes profit and calculate the profit.

EXAMPLE 5 ☑ **Try It!** **Compare Two Polynomial Functions**

5. Compare the profit functions of two additional market sellers modeled by the graph of f and the equation $g(x) = (x + 1)(5 - x)$. Compare and interpret the y-intercepts of these functions and their end behavior.

HABITS OF MIND

Make Sense and Persevere Find the quantity that maximizes profit for $g(x) = (x + 1)(5 - x)$. Calculate the profit.

☑ Do You UNDERSTAND?

1. ❓ **ESSENTIAL QUESTION** How do you add, subtract, and multiply polynomials?

2. Error Analysis Chen subtracted two polynomials as shown. Explain Chen's error.

$$p^2 + 7mp + 4 - (-2p^2 - mp + 1)$$
$$p^2 + 2p^2 + 7mp - mp + 4 + 1$$
$$3p^2 + 6m + 5 \quad ✗$$

3. Communicate Precisely Why do we often write the results of polynomial calculations in standard form?

4. Construct Arguments Is the set of whole numbers closed under subtraction? Explain why you think so, or provide a counterexample.

Do You KNOW HOW?

Add or subtract the polynomials.

5. $(-3a^3 + 2a^2 - 4) + (a^3 - 3a^2 - 5a + 7)$

6. $(7x^2y^2 - 6x^3 + xy) - (5x^2y^2 - x^3 + xy + x)$

Multiply the polynomials.

7. $(7a + 2)(2a^2 - 5a + 3)$

8. $(xy - 1)(xy + 6)(xy - 8)$

9. The length of a rectangular speaker is three times its width, and the height is four more than the width. Write an expression for the volume V of the rectangular prism in terms of its width, w.

PRACTICE & PROBLEM SOLVING

10. **Generalize** Explain two methods by which $\left(2m^3 + 4n^2\right)^2$ can be simplified. Which method do you prefer and why?

11. **Use Structure** Polynomial function P is the sum of two polynomial functions, one with degree 2 and a positive leading coefficient and one with degree 3 and a negative leading coefficient. Describe the end behavior of P. Write an example of two polynomial functions and their sum, P, to justify your description.

12. **Generalize** Multiply the polynomials $(a + b)$ $(a + b)(a + b)$ to develop a general formula for cubing a binomial, $(a + b)^3$.

13. **Reason** Polynomial function R is the difference of two degree-two polynomial functions. What are the possible degrees for R? Explain.

14. **Error Analysis** Describe and correct the error a student made in multiplying the polynomials.

$$(y - 2)(3y^2 - y - 7)$$
$$= y(3y^2 - y - 7) - 2(3y^2 - y - 7)$$
$$= 3y^3 - y^2 - 7y + (- 6y^2) + (- 2y) - 14$$
$$= 3y^3 - 7y^2 - 9y - 14$$

✗

15. **Higher Order Thinking** Do you think polynomials are closed under division? Explain why you think so, or provide a counterexample.

16. **Construct Arguments** Explain why the expression $9x^3 + \frac{1}{2}x^2 + 3x^{-1}$ is not a polynomial.

17. **Communicate Precisely** Explain the difference between the graphs of polynomial functions with a degree of 3 that have a positive leading coefficient and the graphs of those with a negative leading coefficient.

PRACTICE & PROBLEM SOLVING

Add or subtract the polynomials. SEE EXAMPLE 1

18. $(2x^3 + 3x^2 + 4) + (6x^3 - x^2 - 5x)$

19. $(5y^4 + 3y^3 - 6y^2 + 14) - (-y^4 + y^2 - 7y - 1)$

20. $(4p^2q^2 + 2p^2q - 7pq) - (9p^2q^2 + 5pq^2 - 11pq)$

Multiply the polynomials. SEE EXAMPLE 2

21. $-4xy(5x^2 - 9xy - y^2)$

22. $(3c - 4)(2c^2 - 5c + 7)$

23. $(z + 5)(z - 9)(1 - z)$

24. Is the set of monomials closed under addition? Explain why you think so, or provide a counterexample. SEE EXAMPLE 3
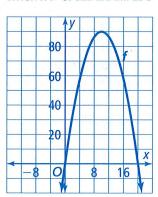
25. An online shopping club has 13,500 members when it charges $8 per month for membership. For each $1 monthly increase in membership fee, the club loses approximately 500 of its existing members.

Write and simplify a function R to represent the monthly revenue received by the club when x represents the price increase.

Hint Monthly revenue = # members • monthly fee SEE EXAMPLE 4

26. The graph shows a polynomial function f. Polynomial function g is defined by $g(x) = x^2(6 - x)$. Compare the maximum values and the end behavior of the functions f and g when $x > 0$. SEE EXAMPLE 5
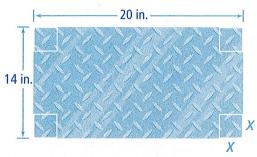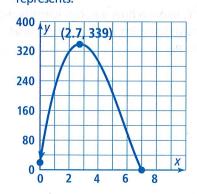
PRACTICE & PROBLEM SOLVING

APPLY

Use this information for 27 and 28. A foundry manufactures aluminum trays from pieces of sheet metal as shown.

27. Model With Mathematics Let x represent the side length of each square.

a. Write expressions for the length, width, and height of the metal tray.

b. Write and simplify a polynomial function V to represent the volume of the tray.

c. Using the graph of the function V, explain what the marked relative maximum represents.

28. Reason Suppose the foundry manufacturer has a new design where the squares cut from the corners have sides that are half the length of the squares in the previous design.

a. Write expressions for the length, width, and height of this tray.

b. Write and simplify the polynomial function $v(x)$, to represent the volume of the new tray.

c. Write the function $D(x)$ that represents the difference, $V(x) - v(x)$.

29. Make Sense and Persevere Jacy has $1,000 to invest in a fund that pays approximately 4.6% per year or in a savings account with an annual interest rate of 1.8%. Write a polynomial function $S(x)$ to represent the interest Jacy will earn in 1 year by investing x dollars in the fund and the remainder in the savings account.

30. Are polynomials open or closed under each operation? Classify each operation as *open* or *closed*.

   a. addition

   b. subtraction

   c. multiplication

   d. division

31. SAT/ACT Which of the following functions is NOT a polynomial function?

Ⓐ $2y^2 + 9y - 8$

Ⓑ $-\frac{1}{2}x^3 + 8$

Ⓒ $(x-1)(5-x)(x+4)$

Ⓓ $9z^4 + 2z + \frac{1}{z}$

32. Performance Task Consider the polynomial functions $P(x) = x^2 - 4$ and $R(x) = -x^2 - 2x$.

Part A Write and simplify a polynomial function $T(x)$ that is the product of P and R.

Part B Complete the table of values for all three functions.

x	P(x)	R(x)	T(x)
−3			
−2			
−1			
0			
1			
2			
3			

Part C Graph the functions on the same coordinate grid.

Part D How do the zeros of T relate to the zeros of P and R?

Part E Explain how you can identify the intervals in which T is positive by analyzing the R and P.

EXPLORE & REASON

Look at the following triangle. Each number is the sum of the two numbers diagonally above. If there is not a second number, think of it as 0.

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5   10   10   5   1
```

A. Write the numbers in the next 3 rows.

B. Look for Relationships What other patterns do you see?

C. Write a formula for the sum of the numbers in the nth row of the triangle.

HABITS OF MIND

Look for Relationships Create a triangle that starts with 2 instead of 1. How does this new triangle relate to the original triangle?

EXAMPLE 1   ☑ **Try It!**   **Prove a Polynomial Identity**

1. Prove the Difference of Cubes Identity.

HABITS OF MIND

Reason Is the trinomial factor in the Difference of Cubes Identity a perfect square trinomial? Explain.

EXAMPLE 2   ☑ **Try It!**   **Use Polynomial Identities to Multiply Polynomials**

2. Use polynomial identities to multiply each expression.

a. $(3x^2 + 5y^3)(3x^2 - 5y^3)$

b. $(12 + 15)^2$

EXAMPLE 3   ☑ **Try It!**   **Use Polynomial Identities to Factor Polynomials**

3. Use polynomial identities to factor each polynomial or simplify each expression.

a. $m^8 - 9n^{10}$

b. $27x^9 - 343y^6$

c. $12^3 + 2^3$

HABITS OF MIND

Look for Relationships What binomial has factors $(a - 3b)$ and $(a^2 + 3ab + 9b^2)$?

EXAMPLE 4 ☑ **Try It!** **Expand a Power of a Binomial**

4. Use Pascal's Triangle to expand $(x + y)^6$.

EXAMPLE 5 ☑ **Try It!** **Apply the Binomial Theorem**

5. Use the Binomial Theorem to expand each expression.

a. $(x - 1)^7$

b. $(2c + d)^6$

HABITS OF MIND

Use Structure For what binomial expression is the expansion
$243x^5 - 405x^4y^2 + 270x^3y^4 - 90x^2y^6 + 15xy^8 - y^{10}$?

✅ Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you use polynomial identities to rewrite expressions efficiently?

2. **Reason** Explain why the middle term of $(x + 5)^2$ is $10x$.

3. **Communicate Precisely** How are Pascal's Triangle and a binomial expansion, such as $(a + b)^5$, related?

4. **Use Structure** Explain how to use a polynomial identity to factor $8x^6 - 27y^3$.

5. **Make Sense and Persevere** What does C_3 represent in the expansion $C_0a^5 + C_1a^4b + C_2a^3b^2 + C_3a^2b^3 + C_4ab^4 + C_5b^5$? Explain.

6. **Error Analysis** Dakota said the third term of the expansion of $(2g + 3h)^4$ is $36g^2h^2$. Explain Dakota's error. Then correct the error.

Do You KNOW HOW?

Use polynomial identities to multiply each expression.

7. $(2x + 8y)(2x - 8y)$

8. $(x + 3y^3)^2$

Use polynomial identities to factor each polynomial.

9. $36a^6 - 4b^2$

10. $8x^6 - y^3$

11. $m^9 + 27n^6$

Find the term of each binomial expansion.

12. fifth term of $(x + y)^5$

13. third term of $(a - 3)^6$

Use Pascal's Triangle to expand each expression.

14. $(x + 1)^5$

15. $(a - b)^6$

Use the Binomial Theorem to expand each expression.

16. $(d - 1)^4$

17. $(x + y)^7$

✎ PRACTICE & PROBLEM SOLVING

UNDERSTAND

18. **Use Structure** Expand $(3x + 4y)^3$ using Pascal's Triangle and the Binomial Theorem.

19. **Error Analysis** Emma factored $625g^{16} - 25h^4$. Describe and correct the error Emma made in factoring the polynomial.

$$625g^{16} - 25h^4$$
$$= (25g^4)^2 - (5h^2)^2$$
$$= (25g^4 + 5h^2)(25g^4 - 5h^2)$$

✗

20. **Higher Order Thinking** Use Pascal's Triangle and the Binomial Theorem to expand $(x + i)^4$. Justify your work.

21. **Use Structure** Expand the expression $(2x - 1)^4$. What is the sum of the coefficients?

22. **Error Analysis** A student says that the expansion of the expression $(-4y + z)^7$ has seven terms. Describe and correct the error the student may have made.

23. **Reason** The sum of the coefficients in the expansion of the expression $(a + b)^n$ is 64. Use Pascal's Triangle to find the value of n.

24. **Use Structure** Factor $x^3 - 125y^6$ in the form $(x - A)(x^2 + Bx + C)$. What are the values of A, B, and C?

25. **Generalize** How many terms will there be in the expansion of the expression $(x + 3)^n$? Explain how you know.

26. **Make Sense and Persevere** How could you use polynomial identities to factor the expression $x^6 - y^6$?

PRACTICE & PROBLEM SOLVING

PRACTICE

27. Prove the polynomial identity.
$$x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$$
SEE EXAMPLE 1

Use polynomial identities to multiply the expressions. SEE EXAMPLE 2

28. $(x + 9)(x - 9)$

29. $(x + 6)^2$

30. $(3x - 7)^2$

31. $(2x - 5)(2x + 5)$

32. $(4x^2 + 6y^2)(4x^2 - 6y^2)$

33. $(x^2 + y^6)^2$

34. $(8 - x^2)(8 + x^2)$

35. $(6 - y^3)^2$

36. $18 \cdot 22$

37. $103 \cdot 97$

38. $(7 + 9)^2$

39. $(10 + 5)^2$

Use polynomial identities to factor the polynomials or simplify the expressions. SEE EXAMPLE 3

40. $x^8 - 9$

41. $x^9 - 8$

42. $8x^3 + y^9$

43. $x^6 - 27y^3$

44. $4x^2 - y^6$

45. $216 + 27y^{12}$

46. $64x^3 - 125y^6$

47. $\frac{1}{16}x^6 - 25y^4$

48. $9^3 + 6^3$

49. $10^3 + 5^3$

50. $10^3 - 3^3$

51. $8^3 - 2^3$

Use the Binomial Theorem to expand the expressions. SEE EXAMPLES 4 and 5

52. $(x + 3)^3$

53. $(2a - b)^5$

54. $\left(b - \frac{1}{2}\right)^4$

55. $(x^2 + 1)^4$

56. $\left(2x + \frac{1}{3}\right)^3$

57. $(x^3 + y^2)^6$

58. $(d - 3)^4$

59. $(2m + 2n)^6$

60. $(n + 5)^5$

61. $(3x - 0.2)^3$

62. $(4g + 2h)^4$

63. $\left(m^2 + \frac{1}{2}n\right)^3$

PRACTICE & PROBLEM SOLVING

APPLY
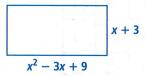
64. Reason A medium-sized shipping box with side length s units has a volume of s^3 cubic units.

a. A large shipping box has side lengths that are 3 units longer than the medium shipping box. Write a binomial expression for the volume of the large shipping box.

b. Expand the polynomial in part a to simplify the volume of the large shipping box.

c. A small shipping box has side lengths that are 2 units shorter than the medium shipping box. Write a binomial expression for the volume of the small shipping box.

d. Expand the polynomial in part c to simplify the volume of the small shipping box.

65. Use Structure The dimensions of a rectangle are shown. Write the area of the rectangle as a sum of cubes.

$x + 3$

$x^2 - 3x + 9$

66. A Pythagorean triple is a set of three positive integers a, b, and c that satisfy $a^2 + b^2 = c^2$. The identity $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$ can be used to generate Pythagorean triples. Use the identity to generate a Pythagorean triple when $x = 5$ and $y = 4$.

✅ ASSESSMENT PRACTICE

67. Are the expressions below perfect square trinomials? Select *Yes* or *No*.

	Yes	No
$x^2 + 16x + 64$	☐	☐
$4x^2 - 44x + 121$	☐	☐
$9x^2 - 15x + 25$	☐	☐

68. SAT/ACT How many terms are in the expansion of $(2x + 7y)^9$?

Ⓐ 2

Ⓑ 7

Ⓒ 8

Ⓓ 9

Ⓔ 10

69. Performance Task If an event has a probability of success p and a probability of failure q, then each term in the expansion of $(p + q)^n$ represents a probability. For example, if a basketball player makes 60% of his free throw attempts, $p = 0.6$ and $q = 0.4$. To find the probability the basketball player will make exactly h out of k free throws, find $C_{k-h}p^h q^{k-h}$, where C_{k-h} is a coefficient of row k of Pascal's Triangle, p is the probability of success, and q is the probability of failure.

$p = 0.6$ $q = 0.4$

Part A What is the probability the basketball player will make exactly 6 out of 10 free throws? Round to the nearest percent.
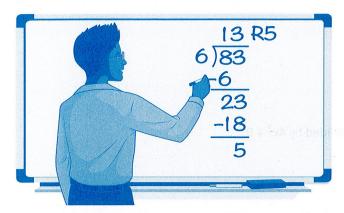
Part B Another basketball player makes 80% of her free throw attempts. Write an expression to find the probability of this basketball player making exactly 7 out of 10 free throws. Describe what each variable in the expression represents.

Part C Find the probability that the basketball player from Part B will make exactly 7 out of 10 free throws. Round to the nearest percent.

EXPLORE & REASON

Benson recalls how to divide whole numbers by solving a problem with 6 as the divisor and 83 as the dividend. He determines that the quotient is 13 with remainder 5.

$$
\begin{array}{r}
13 \text{ R}5 \\
6\overline{)83} \\
-6 \\
\hline
23 \\
-18 \\
\hline
5
\end{array}
$$

A. Explain the process of long division using Benson's example.

B. How can you express the remainder as a fraction?

C. **Use Structure** Use the results of the division problem to write two expressions for 83 that include the divisor, quotient, and remainder.

HABITS OF MIND

Look for Relationships If the remainder in a division problem is zero, what can you say about the dividend?

EXAMPLE 1 ☑ **Try It!** Use Long Division to Divide Polynomials

1. Use long division to divide the polynomials.

  a. $x^3 - 6x^2 + 11x - 6$ divided by $x^2 - 4x + 3$

  b. $16x^4 - 85$ divided by $4x^2 + 9$

EXAMPLE 2 ☑ **Try It!** Use Synthetic Division to Divide by $x - a$

2. Use synthetic division to divide $3x^3 - 5x + 10$ by $x - 1$.

HABITS OF MIND

Communicate Precisely Which method would you use to divide a polynomial by $x^2 + 5$? Why?

EXAMPLE 3 ☑ **Try It!** Relate $P(a)$ to the Remainder of $P(x) \div (x - a)$

3. Use synthetic division to show that the remainder of $f(x) = x^3 + 8x^2 + 12x + 5$ divided by $x + 2$ is equal to $f(-2)$.

EXAMPLE 4 ☑ **Try It!** **Use the Remainder Theorem to Evaluate Polynomials**

4. A technology company uses the function $R(x) = -x^3 + 12x^2 + 6x + 80$ to model expected annual revenue, in thousands of dollars, for a new product, where x is the number of years after the product is released. Use the Remainder Theorem to estimate the revenue in year 5.

EXAMPLE 5 ☑ **Try It!** **Use the Factor Theorem**

5. Use the Remainder and Factor Theorems to determine whether the given binomial is a factor of $P(x)$.

a. $P(x) = x^3 - 10x^2 + 28x - 16$; binomial: $x - 4$

b. $P(x) = 2x^4 + 9x^3 - 2x^2 + 6x - 40$; binomial: $x + 5$

HABITS OF MIND

Make Sense and Persevere Is $x - 2$ a factor of $x^5 + x^4 - 6x^3 + 2x^2 - 11x + 15$? If not, what is the remainder?

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you divide polynomials?

2. Error Analysis Ella said the remainder of $x^3 + 2x^2 - 4x + 6$ divided by $x + 5$ is 149. Is Ella correct? Explain.

3. Look for Relationships You divide a polynomial $P(x)$ by a linear expression $D(x)$. You find a quotient $Q(x)$ and a remainder $R(x)$. How can you check your work?

Do You KNOW HOW?

4. Use long division to divide $x^4 - 4x^3 + 12x^2 - 3x + 6$ by $x^2 + 8$.

5. Use synthetic division to divide $x^3 - 8x^2 + 9x - 5$ by $x - 3$.

6. Use the Remainder Theorem to find the remainder of $2x^4 + x^2 - 10x - 1$ divided by $x + 2$.

7. Is $x + 9$ a factor of the polynomial $P(x) = x^3 + 11x^2 + 15x - 27$? If so, write the polynomial as a product of two factors. If not, explain how you know.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

8. **Reason** Write a polynomial division problem with a quotient of $x^2 - 5x + 7$ and a remainder of 2. Explain your reasoning. How can you verify your answer?

9. **Communicate Precisely** Show that $x - 3$ and $x + 5$ are factors of $x^4 + 2x^3 - 16x^2 - 2x + 15$. Explain your reasoning.

10. **Error Analysis** Alicia divided the polynomial $2x^3 - 4x^2 + 6x + 10$ by $x^2 + x$. Describe and correct the error Alicia made in dividing the polynomials.

$$2x - 6 + \frac{10}{x^2 + x}$$

$$x^2 + x \overline{\smash{)}\, 2x^3 - 4x^2 + 6x + 10}$$
$$\underline{-(2x^3 + 2x^2)}$$
$$-6x^2 + 6x$$
$$\underline{-(-6x^2 - 6x)}$$
$$10$$

11. **Higher Order Thinking** When dividing polynomial $P(x)$ by polynomial $d(x)$, the remainder is $R(x)$. The remainder can also be written as $\frac{R(x)}{d(x)}$. How can you use the degrees of $R(x)$ and $d(x)$ to determine whether you are finished dividing?

12. **Look for Relationships** When dividing polynomial $P(x)$ by polynomial $x - n$, the remainder is 0. When graphing $P(x)$, what is an x-intercept of the graph?

_____

13. **Reason** When dividing $x^3 + nx^2 + 4nx - 6$ by $x + 3$, the remainder is -48. What is the value of n?

14. **Mathematical Connections** Use polynomial long division to divide $8x^3 + 27$ by $2x + 3$. How can you use multiplication to check your answer? Show your work.

✏️ PRACTICE & PROBLEM SOLVING

PRACTICE ▶

Use long division to divide. SEE EXAMPLE 1

15. $x^3 + 5x^2 - x - 5$ divided by $x - 1$

16. $2x^3 + 9x^2 + 10x + 3$ divided by $2x + 1$

17. $3x^3 - 2x^2 + 7x + 9$ divided by $x^2 - 3x$

18. $2x^4 - 6x^2 + 3$ divided by $2x - 6$

Use synthetic division to divide. SEE EXAMPLE 2

19. $x^4 - 25x^2 + 144$ divided by $x - 4$

20. $x^3 + 6x^2 + 3x - 10$ divided by $x + 5$

21. $x^5 + 2x^4 - 3x^3 + x - 1$ divided by $x + 2$

22. $-x^4 + 7x^3 + x^2 - 2x - 12$ divided by $x - 3$

23. Use synthetic division to show that the remainder of $f(x) = x^4 - 6x^3 - 33x^2 + 46x + 75$ divided by $x - 9$ is $P(9)$. SEE EXAMPLE 3

Use the Remainder Theorem to evaluate each polynomial for the given value of x. SEE EXAMPLE 4

24. $f(x) = x^3 + 9x^2 + 3x - 7;\ x = -5$

25. $f(x) = 2x^3 - 3x^2 + 4x + 13;\ x = 3$

26. $f(x) = -x^4 + 2x^3 - x^2 + 4x + 8;\ x = -2$

27. $f(x) = x^5 - 3x^4 - 2x^3 + x^2 - 2x - 1;\ x = 4$

Is each given binomial a factor of the given polynomial? If so, write the polynomial as a product of two factors. SEE EXAMPLE 5

28. polynomial: $P(x) = 8x^3 - 10x^2 + 28x - 16$; binomial: $x - 3$

29. polynomial: $P(x) = 4x^4 - 9x^3 - 7x^2 - 2x + 25$; binomial: $x + 4$

30. polynomial: $P(x) = -x^5 + 12x^3 + 6x^2 - 23x + 1$; binomial: $x - 2$
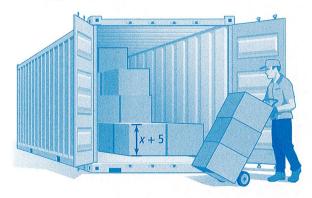
31. polynomial: $P(x) = 2x^3 + 3x^2 - 8x - 12$; binomial: $2x + 3$

✓ ASSESSMENT PRACTICE

35. When polynomial $P(x)$ is divided by the linear factor $x - n$, the remainder is 0. What can you conclude? Select all that apply.

Ⓐ $P(x) = 0$

Ⓑ $P(n) = 0$

Ⓒ $P(-n) = 0$

Ⓓ $x - n$ is a factor of $P(x)$.

Ⓔ $x + n$ is a factor of $P(x)$.

36. SAT/ACT $x + 3$ is a factor of the polynomial $x^3 + 2x^2 - 5x + n$. What is the value of n?

Ⓐ -6

Ⓑ -3

Ⓒ -2

Ⓓ 3

Ⓔ 6

37. Performance Task The table shows some quotients of the polynomial $x^n - 1$ divided by the linear factor $x - 1$.

Dividend	Divisor	Quotient
$x^2 - 1$	$x - 1$	$x + 1$
$x^3 - 1$	$x - 1$	$x^2 + x + 1$
$x^4 - 1$	$x - 1$	
$x^5 - 1$	$x - 1$	
$x^6 - 1$	$x - 1$	

Part A Use long division or synthetic division to find the missing quotients to complete the table.
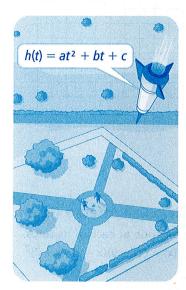
Part B Look for a pattern. Then describe the pattern when $x^n - 1$ is divided by $x - 1$.

Part C Use the pattern to find the quotient when $x^{10} - 1$ is divided by $x - 1$.
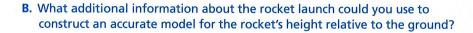
MODEL & DISCUSS

Charlie and Aisha built a small rocket and launched it from their backyard. The rocket fell to the ground 10 s after it launched. The height h, in feet, of the rocket relative to the ground at time t seconds can be modeled by the function shown.

$h(t) = at^2 + bt + c$
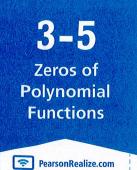
A. How are the launch and landing times related to the modeling function?

B. What additional information about the rocket launch could you use to construct an accurate model for the rocket's height relative to the ground?

C. Construct Arguments Charlie believes that the function $h(t) = -16t^2 + 160t$ models the height of the rocket with respect to time. Do you agree? Explain your reasoning and indicate the domain of this function.

HABITS OF MIND

Reason In Charlie's function, what is the value of c? Why is this the correct value?
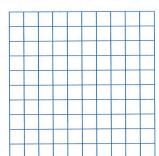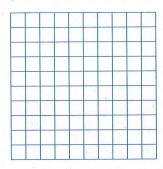
EXAMPLE 1   ☑ **Try It!**   **Use Zeros to Graph a Polynomial Function**

1. Factor each function. Then use the zeros to sketch its graph.

    a. $f(x) = 4x^3 + 4x^2 - 24x$ **b.** $g(x) = x^4 - 81$

EXAMPLE 2   ☑ **Try It!**   **Understand How a Multiple Zero Can Affect a Graph**

2. Describe the behavior of the graph of the function at each of its zeros.

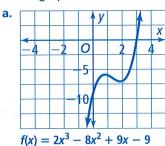   **a.** $f(x) = x(x + 4)(x - 1)^4$ **b.** $f(x) = (x^2 + 9)(x - 1)^5(x + 2)^2$

HABITS OF MIND

Reason Do the values of a function always change from positive to negative or negative to positive on either side of a zero? Explain.
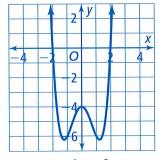
EXAMPLE 3   ☑ **Try It!**   **Find Real and Complex Zeros**

3. What are all the real and complex zeros of the polynomial function shown in the graph?

    a.

$f(x) = 2x^3 - 8x^2 + 9x - 9$

    b.

$f(x) = x^4 - 3x^2 - 4$

EXAMPLE 4 ☑ **Try It!** **Interpret the Key Features of a Graph in Context**

4. Due to a decrease in the cost of materials, the profit function for Acme Innovations has changed to $Q(x) = -x^3 + 10x^2 + 13x - 22$. How many lamps should they make in order to make a profit?

HABITS OF MIND

Make Sense and Persevere On a graph, how do complex roots differ from real roots?

EXAMPLE 5 ☑ **Try It!** **Solve Polynomial Equations**

5. What is the solution of the equation?

a. $x^3 - 7x + 6 = x^3 + 5x^2 - 2x - 24$ b. $x^4 + 2x^2 = -x^3 - 2x$

EXAMPLE 6 ☑ **Try It!** **Solve a Polynomial Inequality by Graphing**

6. What are the solutions of the inequality?

a. $2x^3 + 12x^2 + 12x < 0$ b. $(x^2 - 1)(x^2 - x - 6) > 0$

HABITS OF MIND

Use Structure How does solving $2x^3 + 12x^2 + 12x = 0$ help you to solve the inequality $2x^3 + 12x^2 + 12x < 0$?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are the zeros of a polynomial function related to the equation and graph of a function?

2. **Error Analysis** In order to identify the zeros of the function, a student factored the cubic function $f(x) = x^3 - 3x^2 - 10x$ as follows:
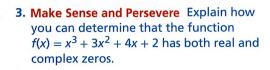
$$f(x) = x^3 - 3x^2 - 10x$$
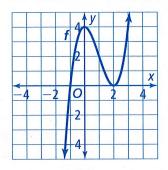$$= x(x^2 - 3x - 10)$$
$$= x(x - 5)(x + 2)$$
$$x = 0, x = -5, x = 2$$

Describe and correct the error the student made.

3. **Make Sense and Persevere** Explain how you can determine that the function $f(x) = x^3 + 3x^2 + 4x + 2$ has both real and complex zeros.

Do You KNOW HOW?

4. If the graph of the function f has a multiple zero at $x = 2$, what is a possible exponent of the factor $x - 2$? Justify your reasoning.

5. Energy Solutions manufactures LED light bulbs. The profit p, in thousands of dollars earned, is a function of the number of bulbs sold, x, in ten thousands. Profit is modeled by the function $-x^3 + 9x^2 - 11x - 21$. For what number of bulbs manufactured is the company profitable?
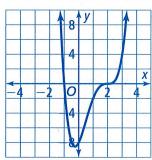
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PRACTICE & PROBLEM SOLVING
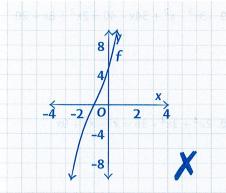
UNDERSTAND

6. **Reason** If you use zeros to sketch the graph of a polynomial function, how can you verify that your graph is correct?

7. **Error Analysis** Describe and resolve two errors that Tonya may have made in finding all the roots of the polynomial function, $f(x) = x^3 + 3x^2 + 7x + 5$.

This function has only one real root at $x = -1$.

8. **Higher Order Thinking** How could you use your graphing calculator to determine that $f(x) = (x + 2)(x + 6)(x - 1)$ is not the correct factorization of $f(x) = x^3 + 7x^2 + 16x + 12$? Explain.

9. **Generalize** How can you determine that the polynomial function shown does not have any zeros with even multiplicity? Explain.

10. **Use Structure** Factor the polynomial $x^4 - 16$. How many real zeros does the function $g(x) = x^4 - 16$ have?

11. At what points do the graphs of $f(x) = x^3 - 2x^2 - 16x + 20$ and $g(x) = -12$ intersect?

PRACTICE & PROBLEM SOLVING

PRACTICE ▶

Sketch the graph of the function by finding the zeros. SEE EXAMPLE 1

12. $f(x) = 3x^3 - 9x^2 - 12x$
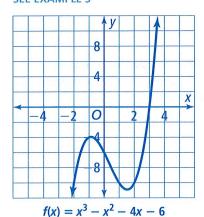
13. $g(x) = (x + 3)(x - 1)(x - 4)$

Find the zeros of the function, and describe the behavior of the graph at each zero. SEE EXAMPLE 2

14. $f(x) = x^3 - 8x^2 + 16x$

15. $g(x) = x^3 - x^2 - 25x + 25$

16. $f(x) = 9x^4 - 40x^2 + 16$

17. What are all the real and complex zeros of the polynomial function shown in the graph?
SEE EXAMPLE 3

$f(x) = x^3 - x^2 - 4x - 6$

18. Waterworks is a company that manufactures and sells paddleboards. Their profit P, in hundreds of dollars earned, is a function of the number of paddleboards sold x, measured in thousands. Profit is modeled by the function $P(x) = -3x^3 + 48x^2 - 144x$. What do the zeros of the function tell you about the number of paddleboards that Waterworks should produce? SEE EXAMPLE 4

What are the solution(s) of the equation?
SEE EXAMPLE 5

19. $-3x^3 - x^2 + 54x - 40 = 2x^2 + 6x + 20$

20. $2x^3 + 3x^2 - 36 = x^3 - x^2 + 9x$

21. $-5x^4 + 4x^2 - 12x = -6x^4 + 3x^3$

What are the solutions of the inequality?
SEE EXAMPLE 6

22. $x^3 - 9x > 0$ **23.** $0 > 4x^3 + 8x^2 - x - 2$

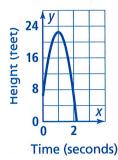**24.** $64x^2 > -4x^3 - x - 16$

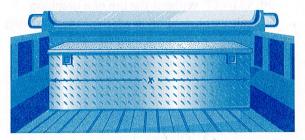# PRACTICE & PROBLEM SOLVING

APPLY

25. Make Sense and Persevere A firework is launched vertically into the air. Its height in meters is given by the function shown, where t is measured in seconds.

$$h = -4.9t^2 + 49t$$

a. What is a reasonable domain of the function?

b. What are the zeros of the function? Explain what they represent in this situation.

c. Use technology to find the vertex. What does it represent in this situation?

26. The height of a baseball thrown in the air can be modeled by the function $h(t) = -16t^2 + 32t + 6.5$, where $h(t)$ represents the height in feet of the baseball after t seconds. Explain why the graph of this function only shows one zero.

27. Model With Mathematics The height of a rectangular storage box is less than both its length and width. The function $f(x) = x^3 + 2x^2 - 3x$ represents the volume of the rectangular box, where x represents the width of the box, in feet.

a. Find the factored form of $f(x)$.

b. Find the zeros of the function.

c. You know x represents the width of the box. What do the other two factors represent?

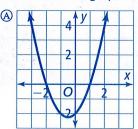**d.** Find the dimensions of the box when the volume is 10 ft^3.
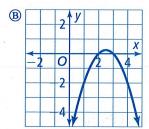
ASSESSMENT PRACTICE

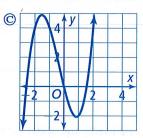**28.** Complete each statement so it means the same as 4 is *a zero of the function*.
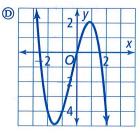
The graph of the function crosses the

_____ at 4. _____ is a factor of the polynomial.

29. SAT/ACT Without the use of a graphing calculator, determine which of the following functions is the graph of $f(x) = x^3 + x^2 - 4x$.

Ⓐ

Ⓑ

Ⓒ

Ⓓ

30. Performance Task Venetta opened several deli sandwich franchises in 2000. The profit P (in hundreds of dollars) of the franchises in t years (since the franchises opened) can be modeled by the function $P(t) = t^3 + t^2 - 6t$.

Part A Sketch a graph of the function.

Part B Based on the model, during what years did Venetta not make a profit?

Part C If the model is appropriate, predict the amount of profit Venetta will receive from her franchises in 2020.

What Are the Rules?

All games have rules about how to play the game. The rules outline such things as when a ball is in or out, how a player scores points, and how many points a player gets for each winning shot.

If you didn't already know how to play tennis, or some other game, could you figure out what the rules were just by watching? What clues would help you understand the game? Think about this during the Mathematical Modeling in 3 Acts lesson.

MATHEMATICAL MODELING IN 3 ACTS

PearsonRealize.com

ACT 1 **Identify the Problem**

1. What is the first question that comes to mind after watching the video?

2. Write down the Main Question you will answer.

3. Make an initial conjecture that answers this Main Question.

4. Explain how you arrived at your conjecture.

5. What information will be useful to know to answer the Main Question? How can you get it? How will you use that information?

ACT 2 ▶ Develop a Model

6. Use the math that you have learned in the topic to refine your conjecture.

ACT 3 ▶ Interpret the Results

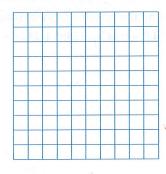**7.** Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

CRITIQUE & EXPLAIN

Look at the polynomial functions shown.

$$g(x) = x^2 - 7x - 18$$
$$h(x) = 5x^2 + 24x + 16$$

A. Avery has a conjecture that the zeros of a polynomial function have to be positive or negative factors of its constant term. Factor g completely. Are the zeros of g factors of -18?

B. Look for Relationships Now test Avery's conjecture by factoring $h(x)$. Does Avery's conjecture hold? If so, explain why. If not, make a new conjecture.

HABITS OF MIND

Use Structure For the factored function $k(x) = (ax + b)(cx + d)$, what are the coefficients in the expanded expression?

Assess

EXAMPLE 1 ☑ **Try It!** **Identify Possible Rational Solutions**

1. List all the possible rational solutions for each equation.

a. $4x^4 + 13x^3 - 124x^2 + 212x - 8 = 0$

b. $7x^4 + 13x^3 - 124x^2 + 212x - 45 = 0$

EXAMPLE 2 ☑ **Try It!** **Use the Rational Root Theorem**

2. A jewelry box measures $2x + 1$ in. long, $2x - 6$ in. wide, and x in. tall. The volume of the box is given by the function $v(x) = 4x^3 - 10x^2 - 6x$. What is the height of the box, in inches, if its volume is 28 in.^{3}?

HABITS OF MIND

Critique Arguments For the jewelry box, a student thought that the rational roots could be ± 6, ± 3, ± 2, ± 1, $\pm\frac{3}{2}$, $\pm\frac{1}{2}$, $\pm\frac{1}{4}$, using factors of -6 for the numerator and factors of 4 for the denominator of the possible rational roots. Is the student correct? Explain.

EXAMPLE 3 ☑ **Try It!** **Find All Complex Roots**

3. What are all the complex roots of the equation $x^3 - 2x^2 + 5x - 10 = 0$?

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EXAMPLE 4 ☑ **Try It!** **Irrational Roots and the Coefficients of a Polynomial**

4. Suppose a quadratic polynomial function f has two complex zeros that are a conjugate pair, $a - bi$ and $a + bi$ (where a and b are real numbers). Are all the coefficients of f real? Explain.

HABITS OF MIND

Construct Arguments Could a polynomial equation with rational coefficients have two complex roots that are not conjugates as its only roots? Explain.

EXAMPLE 5 ☑ **Try It!** **Write Polynomial Functions Using Conjugates**

5. a. What is a quadratic equation in standard form with rational coefficients that has a root of $5 + 4i$?

b. What is a polynomial function Q of degree 4 with rational coefficients such that $Q(x) = 0$ has roots $2 - \sqrt{3}$ and $5i$?

HABITS OF MIND

Reason Is it possible to write a polynomial function of degree 3 that has rational coefficients and zeros $2 - \sqrt{3}$ and $5i$? Explain.

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are the roots of a polynomial equation related to the coefficients and degree of the polynomial?

2. **Error Analysis** Renaldo said that a polynomial equation with real coefficients that has zeros $-1 + 2i$ and $3 + \sqrt{5}$ and has a degree of 4. Is Renaldo correct? Explain.

3. **Use Structure** A fifth degree polynomial $P(x)$ with rational coefficients has zeros $2i$ and $\sqrt{7}$. What other zeros does $P(x)$ have? Explain.

4. **Construct Arguments** If one root of a polynomial equation with real coefficients is $4 + 2i$, is it certain that $4 - 2i$ is also a root of the equation? Explain.

Do You KNOW HOW?

List all the possible rational solutions for each equation according to the Rational Roots Theorem. Then find all of the rational roots.

5. $0 = x^3 + 4x^2 - 9x - 36$

6. $0 = x^4 - 2x^3 - 7x^2 + 8x + 12$

7. $0 = 4x^3 + 8x^2 - x - 2$

8. $0 = 9x^4 - 40x^2 + 16$

A polynomial equation with rational coefficients has the given roots. List two more roots of each equation.

9. $1 + \sqrt{11}$ and $-3 + \sqrt{17}$

10. $5 + 12i$ and $-9 - 7i$

11. $12 + 5i$ and $6 - \sqrt{13}$

12. $5 - 15i$ and $17 + \sqrt{23}$

PRACTICE & PROBLEM SOLVING

UNDERSTAND

13. Construct Arguments Consider the polynomial $P(x) = 5x^3 + ms^2 + nx + 6$, where m and n are rational coefficients. Is 3 *sometimes, always,* or *never* a root? Explain.

14. Use Structure Write a fourth-degree polynomial function Q with roots -1, 0, and $2i$.

15. Error Analysis A student says that a fifth-degree polynomial equation with rational coefficients has roots -5, -3, 1, 2, and $\sqrt{3}$. Describe possible errors the student may have made.

16. Reason Write a third-degree polynomial with rational coefficients that has the following possible roots. Explain your reasoning.

$$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{5}{1}, \pm\frac{5}{2}, \pm\frac{10}{1}, \pm\frac{10}{2}$$

17. Error Analysis Describe and correct the error a student made in finding the roots of the polynomial equation $2x^3 - x^2 - 10x + 5 = 0$.

> List all possible rational roots.
>
> $\pm 1, \pm\frac{1}{2}, \pm 5, \pm\frac{5}{2}$
>
> Testing reveals that $\frac{1}{2}$ is a root.
> Dividing the polynomial by the binomial $x - \frac{1}{2}$ results in the factored form
>
> $f(x) = (x - \frac{1}{2})(2x^2 - 10)$
>
> The equation $2x^2 - 10 = 0$ has two irrational roots, $\sqrt{10}$ and $-\sqrt{10}$.
>
> The complete set of roots is $\{\frac{1}{2}, \sqrt{10}, -\sqrt{10}\}$. ✗

18. Higher Order Thinking What is the least number of terms a fifth-degree polynomial with root $3i$ can have? Give an example of such a polynomial equation. Explain.

19. Use Structure Show that the Fundamental Theorem of Algebra is true for all quadratic equations with real coefficients. (*Hint:* Use the Quadratic Formula and examine the possibilities for the value of the discriminant.)

✎ PRACTICE & PROBLEM SOLVING

▸ PRACTICE

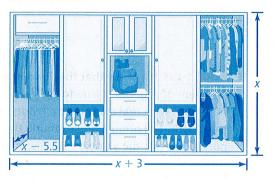**List all the possible rational solutions for each equation.** SEE EXAMPLE 1

20. $0 = x^3 - 3x^2 + 4x - 12$

21. $0 = 2x^4 + 13x^3 - 47x^2 - 13x + 45$

22. $0 = 4x^3 + 64x^2 - x - 16$

23. $0 = 8x^3 + 11x^2 - 13x - 6$

24. A closet in the shape of a rectangular prism has the measurements shown. What is the height of the closet, in feet, if its volume is 220 ft³? SEE EXAMPLE 2

What are all real and complex roots of the following functions? SEE EXAMPLE 3

25. $0 = x^3 - 3x - 52$

26. $0 = x^3 + 9x^2 - 7x - 63$

27. $0 = x^4 + 34x^2 - 72$

28. $0 = x^6 + 4x^4 - 41x^2 + 36$

29. Suppose a cubic polynomial f has one rational zero c and two irrational zeros which are a conjugate pair $a + \sqrt{b}$ and $a - \sqrt{b}$, where a and b are rational numbers. Does f have rational coefficients? SEE EXAMPLE 4

Find a polynomial function $P(x)$ such that P has the degree and $P(x) = 0$ has the root(s) listed. SEE EXAMPLE 5

30. degree of $P = 2$;
zero: $1 + 6i$

31. degree of $P = 4$;
zeros: $3 - \sqrt{11}$ and $-9i$

32. degree of $P = 3$;
zeros: -5 and $4 - 8i$

PRACTICE & PROBLEM SOLVING
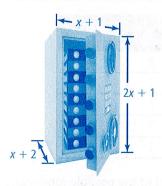
APPLY

33. Make Sense and Persevere A fireproof safe has the measurements shown.
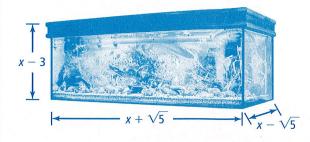

- $x + 1$
- $2x + 1$
- $x + 2$

a. Write an equation to represent the situation when the volume of the fireproof safe is 270 in.^{3}. Rewrite the equation in the form $P(x) = 0$.

b. List all of the possible factors of the polynomial expression.

c. What are the real roots of the equation? Explain how you know these are the only real roots.

d. What are the length, width, and height of the fireproof safe?

34. Reason What are the dimensions of the fish tank, in feet, if its volume is 176 ft^3?

- $x - 3$
- $x + \sqrt{5}$
- $x - \sqrt{5}$

35. Reason The cost of producing x video game consoles is modeled by the function $C(x) = x^4 - 5x^3 - 12x^2 - 22x - 40$. If a company spent \$1,706 to produce video game consoles, how many consoles were made?

☑ **ASSESSMENT PRACTICE**

36. A fifth-degree polynomial equation with rational coefficients has the roots 3, 8i, and 7 − $\sqrt{5}$. Which are also roots of the polynomial equation? Select all that apply.

Ⓐ −3

Ⓑ −8i

Ⓒ 1 − 8i

Ⓓ −7 − $\sqrt{5}$

Ⓔ 7 + $\sqrt{5}$

37. SAT/ACT Which is a third-degree polynomial equation with rational coefficients that has roots −2 and 6i?

Ⓐ $x^3 + 2x^2 + 36x + 72$

Ⓑ $x^3 − 2x^2 + 36x − 72$

Ⓒ $x^3 + 2x^2 − 36x − 72$

Ⓓ $x^2 + (6i − 2)x − 12$

Ⓔ $x^2 − (6i − 2)x − 12$

38. Performance Task The table shows the number of possible real and imaginary roots for an nth degree polynomial equation with rational coefficients.

Degree	Real Roots	Imaginary Roots
3	3	0
3	1	2
5	5	0
5	3	2
5	1	4

Part A List all of the possible combinations of real and imaginary roots for a seventh-degree polynomial equation.
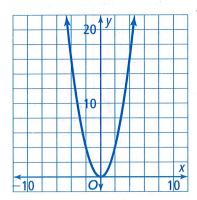
Part B What do you notice about the number of real roots of a polynomial equation with an odd degree?
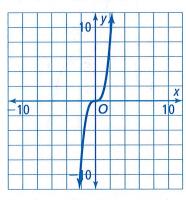
EXPLORE & REASON

Look at the polynomial graphs below.

$f(x) = x^2$

$g(x) = x^3$

A. Is the graph of *f* or *g* symmetric about the *y*-axis? Is the graph of *f* or *g* symmetric about the origin? Explain.
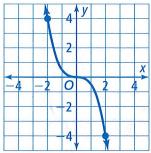
B. **Look for Relationships** Graph more functions of the form $y = x^n$ where *n* is a natural number. Which of these functions are symmetric about the origin? Which are symmetric about the *y*-axis? What conjectures can you make?
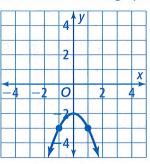
HABITS OF MIND

Look for Relationships Do you notice any other patterns among the functions with even degree or the functions with odd degree?

📶 ☑ Assess

EXAMPLE 1 ☑ **Try It! Identify Even and Odd Functions From Their Graphs**

1. Classify the polynomial functions as even or odd based on the graphs.

a.

b.

EXAMPLE 2 ☑ **Try It! Identify Even and Odd Functions From Their Equations**

2. Is the function odd, even, or neither?

a. $f(x) = 7x^5 - 2x^2 + 4$ b. $f(x) = x^6 - 2$

HABITS OF MIND

Make Sense and Persevere Why do you replace x with $-x$ when determining if a function is odd, even, or neither?

EXAMPLE 3 ☑ **Try It! Graph Transformations of Cubic and Quartic Parent Functions**
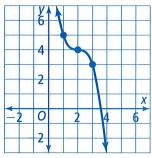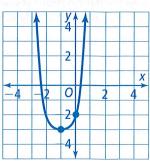
3. How does the graph of the function $g(x) = 2x^3 - 5$ differ from the graph of its parent function?

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EXAMPLE 4 ☑ **Try It! Identify a Transformation**

4. Determine the equation of each graph as it relates to its parent cubic function or quartic function.

a.

b.

HABITS OF MIND

Look for Relationships What type of transformation would change a function's end behavior?

EXAMPLE 5 ☑ **Try It! Apply a Transformation of a Cubic Function**

5. a. The volume of a cube, in cubic feet, is given by the function $V(x) = x^3$. Write a function for the volume of the cube in cubic inches if x is the edge length in feet.

b. A storage unit is in the shape of a rectangular prism. The volume of the storage unit is given by $V(x) = (x)(x)(x - 1) = x^3 - x^2$, where x is measured in feet. A potential customer wants to compare the volume of this storage unit with that of another storage unit that is 1 foot longer in every dimension. Write a function for the volume of this larger unit.
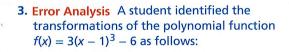
HABITS OF MIND

Critique Arguments A student thought that, for 5a, the new function should be $V(x) = 144x^3$. What are the two errors the student made?

Do You UNDERSTAND?

1. ❓ **ESSENTIAL QUESTION** How are symmetry and transformations represented in the graph and equation of a polynomial function?

2. **Vocabulary** What is the difference between the graph of an even function and the graph of an odd function?
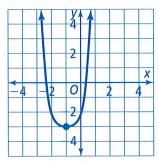
3. **Error Analysis** A student identified the transformations of the polynomial function $f(x) = 3(x - 1)^3 - 6$ as follows:

   The function is shifted to the left 1 unit, stretched vertically, and is shifted downward 6 units.

   Describe and correct the error the student made.

Do You KNOW HOW?

4. Classify the function on the graph as odd, even, or neither.

5. Use the equation to classify the function as odd, even, or neither.

   $g(x) = 4x^3 - x$
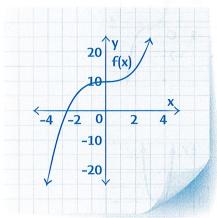
6. The volume of cardboard box is given by the function $V(x) = x(x - 2)(x) = x^3 - 2x^2$. Write a new function for the volume of a cardboard box that is 2 units longer in every dimension.
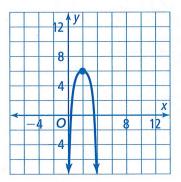
PRACTICE & PROBLEM SOLVING

7. Make Sense and Persevere If you use a graph to determine the equation of a function, explain how to check that your equation is correct.

8. Error Analysis Describe the error Terrence made in graphing the transformation of the cubic function $g(x) = x^3$ to $f(x) = -\frac{1}{2}x^3 + 10$.

9. Higher Order Thinking Explain how to identify a transformation of the function $y = x^3$ by looking at a graph. What do you look for to determine a translation? A reflection? A stretch or compression?

10. Use Structure Describe the steps used to determine the equation of the graph of the transformed parent quartic function.

11. Construct Arguments Explain why the function $g(x) = 2x^5 + 3x^4 + 1$ is neither even nor odd.

12. Construct Arguments Provide an example that demonstrates the following statement is not true.

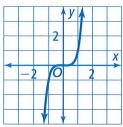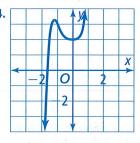*If the degree of a function is an even number, then the function is an even function.*

PRACTICE & PROBLEM SOLVING

PRACTICE

Use the graph to classify the polynomial function. Is it even, odd, or neither? SEE EXAMPLE 1
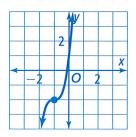
13.

14.

Use the equation to classify the polynomial function. Is it even, odd, or neither? SEE EXAMPLE 2
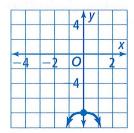
15. $f(x) = 2x^5 + 4x^2$

16. $g(x) = 6x^4 + 2x^2$

How do the graphs of transformations compare to the graph of the parent function? SEE EXAMPLE 3
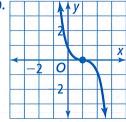
17. $f(x) = 3(x + 1)^3 - 2$

18. $g(x) = -x^4 - 8$

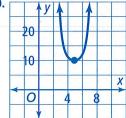**Each graph is a transformation of the parent cubic function or quartic function. Determine the equation of the graph.** SEE EXAMPLE 4

19.

20.

21. The volume of a rectangular room, in cubic yards, is given by the function shown. Write a function for the volume in cubic feet if x is in yards. SEE EXAMPLE 5
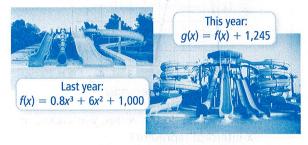

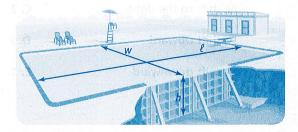Volume in cubic yards: $V(x) = x(3x)(x + 4) = 3x^3 + 12x^2$

PRACTICE & PROBLEM SOLVING

APPLY

22. Make Sense and Persevere Last season the approximate number of guests in week x at an amusement park could be modeled by the function, f, where x represents the number of weeks since the park opened for the season. This year, since the park opened its new water slide, the approximate number of guests in week x at the park can be modeled by g.

This year:
$g(x) = f(x) + 1,245$

Last year:
$f(x) = 0.8x^3 + 6x^2 + 1,000$

a. Write the function g in terms of x.

b. Describe the transformation of the graph of g compared to f.

c. Compare the number of weekly visitors from last year to this year.

23. Generalize The volume of a storage box, in cubic feet, is given by the function $V(x) = (x)(x + 1)^2$. A freight company lists the shipping rates of items in cubic inches. Write a function for the volume of the box in cubic inches if x is its width in feet.

24. Model With Mathematics A swimming pool is in the shape of a rectangular prism. The width is one more than five times the height, and the length is one less than eleven times the height.

a. Using x for the height, write a function $V(x)$ to represent the volume of the pool.

b. Compare the volume of this pool with a larger one that is the same height, but twice the length and twice the width of this pool. Write a function $Z(x)$ for the volume of this larger pool.

✅ ASSESSMENT PRACTICE

25. Match the number in each function with its effect on the parent function.

$f(x) = 2(x - 1)^4 + 5$

$g(x) = (x + 3)^6 - 7$

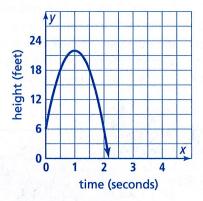|		
I vertical stretch	**A.** 7
II shift to the left	**B.** 5
III shift to the right	**C.** 3
IV shift upward	**D.** 2
V shift downward	**E.** 1

26. SAT/ACT Which of the following functions is neither even nor odd?

Ⓐ $f(x) = x^4 + 3x^2$

Ⓑ $g(x) = 5x^3 - x$

Ⓒ $h(x) = x^5 + 4x^3 + x^2$

Ⓓ $k(x) = 9 - 8x^2$

Ⓔ $p(x) = 5$

27. Performance Task The height of a ball thrown in the air can be modeled by the function $h(x) = -16t^2 + 32t + 6$, where $h(x)$ represents the height in feet of the ball after t seconds. The graph of this function is shown below.

Part A What do the vertex, y-intercept, and x-intercept represent?

Part B If the ball is thrown from a height of 10 ft, how will this transform the graph?

Part C About how much longer will the ball be in the air when it is thrown from 10 ft compared to when it was thrown from 6 ft? (Hint: You may want to use your graphing calculator to compare the two graphs.)