

EXPLORE & REASON

The table represents A(x), the area of a square as a function of side length x units, where x is a positive real number.

Side Length (units)	X	1	2	3	4
Model	x x		\blacksquare		
Area (sq. units)	A(x)	1	4	9	

A. Consider the function where the areas in the table are doubled. Write the equation of a function that represents this.

B. Look for Relationships Graph the ordered pairs for both A(x) and your new function. How would you describe the differences in the locations of these points?

C. Find the equation for a function whose x-values are the same as A(x) but whose y-values are 2 units greater than each y-value in A(x).

HABITS OF MIND

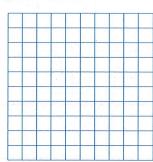
Communicate Precisely Do you think the effect of altering A(x) in the two ways you did would work for any function you start with? Explain.



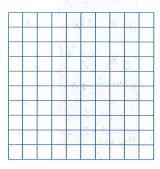


EXAMPLE 1 Try It! Transform a Quadratic Function

- 1. Describe the transformations of the parent function $f(x) = x^2$. Then graph the function.
 - a. $g(x) = -(x+2)^2$



b. $g(x) = (x-1)^2 + 2$



EXAMPLE 2 Try It! Determine Key Features of a Quadratic Function

2. Identify the vertex, axis of symmetry, minimum or maximum, domain, and range of the function $f(x) = -(x + 4)^2 - 5$.

HABITS OF MIND

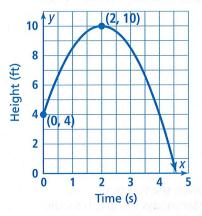
Make Sense and Persevere In what order should you apply the transformations shown in the Try It! for Example 2?

Try It! Write an Equation of a Parabola **EXAMPLE 3**

3. What is the equation of a parabola with a vertex of (1, -4) and which passes through (-2, -1)?

EXAMPLE 4 Try It! Write an Equation of a Parabola Given the Graph

4. The graph shows the height of the flying disk with respect to time. What is the equation of the function? Write the equation in vertex form. Then write the equation in the form $y = ax^2 + bx + c$.



Try It! Write an Equation of a Transformed Function **EXAMPLE 5**

- 5. What is the equation of j? Write the equation in vertex form and in the form $y = ax^2 + bx + c$.
 - a. Let *j* be a quadratic function whose graph is a translation 2 units right and 5 units down of the graph of f.
- **b.** Let *j* be a function whose graph is a reflection of the graph of f in the x-axis followed by a translation 1 unit down.

HABITS OF MIND

Generalize What information do you need to write the equation of a transformed quadratic function in vertex form?





Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How does the equation of a quadratic function in vertex form highlight key features of the function's graph?

2. Error Analysis Given the function $g(x) = (x + 3)^2$, Martin says the graph should be translated right 3 units from the parent graph $f(x) = x^2$. Explain his error.

3. Vocabulary What shape does a quadratic function have when graphed?

4. Communicate Precisely How are the graphs of $f(x) = x^2$ and $g(x) = -(x + 2)^2 - 4$ related?

Do You KNOW HOW?

Describe the transformation of the parent function $f(x) = x^2$.

5.
$$g(x) = -(x+5)^2 + 2$$

6.
$$h(x) = (x+2)^2 - 7$$

Write the equation of each parabola in vertex

8. Vertex: (1, 3); Point: (2, 5)

9. Vertex: (-4, 6); Point: (-2, -2)

10. Vertex: (7, 4); Point: (5, 16)

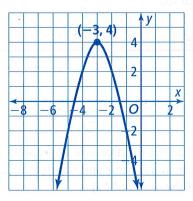




UNDERSTAND

- 11. Use Structure The graph of the function $f(x) = x^2$ will be translated 3 units up and 1 unit left. What is the resulting function g(x)?
- **14. Construct Arguments** Explain why the graph of the equation $g(x) = -(x + 1)^2 3$ would be a parabola opening downward.

- 12. Error Analysis A classmate said that the vertex of $g(x) = -5(x + 2)^2 4$ is (2, 4). Is your classmate correct? If not, what is the correct vertex?
- 15. Use Structure Amaya is standing 30 ft from a volleyball net. The net is 8 ft high. Amaya serves the ball. The path of the ball is modeled by the equation $y = -0.02(x 18)^2 + 12$, where x is the ball's horizontal distance in feet from Amaya's position and y is the distance in feet from the ground to the ball.
- **13. Higher Order Thinking** The graph below is a transformation of the graph of the parent function. Write the quadratic function to model the graph.
- a. How far away is the ball from Amaya when it is at its maximum height? Explain.



b. Describe how you would find the ball's height when it crosses the net at x = 30.





PRACTICE

Describe the transformation of the parent function $f(x) = x^2$. Then graph the transformed function.

SEE EXAMPLE 1

16.
$$f(x) = (x-1)^2 + 3$$
 17. $y = (x+1)^2 - 3$

17.
$$y = (x + 1)^2 - 3$$

18.
$$g(x) = 2x^2$$

19.
$$f(x) = -(x-1)^2 + 7$$

20.
$$y = -2(x+1)^2 + 1$$

20.
$$y = -2(x+1)^2 + 1$$
 21. $f(x) = \frac{1}{2}(x-2)^2 + 3$

Identify the vertex, axis of symmetry, maximum or minimum, domain, and range of each function.

SEE EXAMPLE 2

22.
$$v = 2(x-2)^2 + 5$$

22.
$$y = 2(x-2)^2 + 5$$
 23. $f(x) = -(x-1)^2 + 2$

24.
$$g(x) = -(x+4)^2$$

24.
$$g(x) = -(x+4)^2$$
 25. $y = \frac{1}{3}(x+2)^2 - 1$

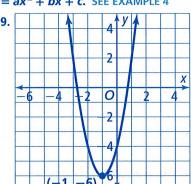
Write the equation of each parabola in vertex form. SEE EXAMPLE 3

26. Vertex: (1, 2); Point: (2, -5)

27. Vertex: (3, 6); y-intercept: 2

28. Vertex: (0, 5); Point: (1, -2)

Write the equation of the function represented by the parabola in vertex form and in the form $y = ax^2 + bx + c$. SEE EXAMPLE 4



Write the equation g(x) in vertex form of a quadratic function for the transformations given the function $f(x) = x^2$. SEE EXAMPLE 5

30. Let g(x) be the function whose graph is a translation 4 units left and 1 unit up of the graph of f(x).

31. Let g(x) be the function whose graph is a reflection in the x-axis and translated 3 units right of the graph of f(x).





APPLY

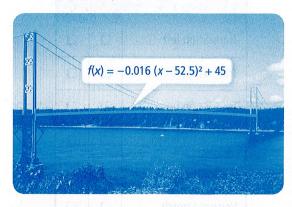
32. Look for Relationships The height, in inches, that a person can jump while wearing a pair of jumping shoes is based on the time, x, in seconds, from the start of the jump. Beth is testing out Max Jumps and Jumpsters to determine which shoes she likes better. Compare the maximum heights on the two sets of shoes.



 $f(x) = -192(x - 0.289)^2 + 16$ $q(x) = -192(x - 0.445)^2 + 38$

33. Make Sense and Persevere Find three additional points on the parabola that has vertex (1, -2) and passes through (0, -5).

34. Make Sense and Persevere The curvature of the Tacoma Narrows Bridge in Washington is in the shape of a parabola.



In the given function, x represents the horizontal distance (in meters) from the arch's left end and f(x) represents the distance (in meters) from the base of the arch. What is the width of the arch?

35. Model With Mathematics An object is thrown from a height of 5 in. After 2 s, the object reaches a maximum height of 9 in., and then it lands back on the ground 5 s after it was thrown. Write the vertex form of the quadratic equation that models the object's path, and draw the graph.



ASSESSMENT PRACTICE

36. The graph of $g(x) = 3(x-2)^2$ is a transformation of the graph of $f(x) = x^2$. Are the following transformations of f that map to g? Select yes or no.

	Yes	No
Translation left		
Translation right	0	
Translation up		0
Translation down		۵
Vertical Compression	٥	
Vertical Stretch		

37. SAT/ACT Which of the following functions represents a parabola with a vertex at (-3, 4) and that passes through the point (-1, -4)?

$$Bf(x) = -2(x+3)^2 + 4$$

©
$$f(x) = 2(x+1)^2 - 4$$

38. Performance Task The Bluebird Bakery sells more cookies when it lowers its prices, but this also changes profits.



The profit function for the cookies is $f(x) = -500(x - 0.45)^2 + 400$. This function represents the profit earned when the price of a cookie is x dollars. The bakery wants to maximize their profits.

Part A What is the domain of the function?

Part B Find the daily profits for selling cookies for \$0.40 each and for \$0.75 each.

Part C What price should the bakery charge to maximize their profits from selling cookies?

Part D What is the maximum profit?





2-2

Standard Form of a Quadratic **Function**

PearsonRealize.com

CRITIQUE & EXPLAIN

Jordan and Emery are rewriting the vertex form of the quadratic function $y = 2(x - 4)^2 + 5$ in the form $y = ax^2 + bx + c$.

Jordan Emery

$$y = 2(x-4)^2 + 5$$
 $y = 2(x-4)^2 + 5$
 $= (2x-8)^2 + 5$ $= 2(x^2-16) + 5$
 $= 4x^2 - 32x + 64 + 5$ $= 2x^2 - 32 + 5$
 $= 4x^2 - 32x + 69$ $= 2x^2 - 27$

A. Communicate Precisely Did Jordan rewrite the equation correctly? Did Emery? Explain.

B. Without rewriting the equation, how could you prove that Jordan's or Emery's equation is not equivalent to the original?

HABITS OF MIND

Reason Casey rewrote the vertex form, too.

$$y = 2(x - 4)^{2} + 5$$

$$= 2(x + 1)^{2}$$

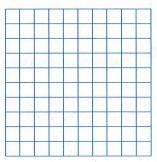
$$= 2(x^{2} + 2x + 1)$$

$$= 2x^{2} + 4x + 2$$

Is Casey correct? Explain.



- Try It! Find the Vertex of a Quadratic Function in **EXAMPLE 1 Standard Form**
 - 1. What is the vertex of the graph of the function $f(x) = x^2 8x + 5$?
- **EXAMPLE 2** Try It! Graph a Quadratic Function in Standard Form
 - 2. Use key features to graph the function $f(x) = x^2 - 6x - 1$.

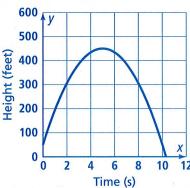


HABITS OF MIND

Error Analysis Yuson said that for the quadratic function $f(x) = 2x^2 + 3x + 1$, the vertex is at the point (0, 1). Is she correct? Explain.

EXAMPLE 3 Try It! Interpret the Graph of a Quadratic Function

3. A water balloon was thrown from a window. The height of the water balloon over time can be modeled by the function $y = -16x^2 + 160x + 50$. What was the maximum height of the water balloon after it was thrown?



HABITS OF MIND

Make Sense and Persevere How long did it take for the water balloon to reach its maximum height?



4. What is the equation of a parabola that passes through the points (2, -12), (-1, -15), and (-4, -90)?

Try It! Use Quadratic Regression EXAMPLE 5

5. A fan threw a souvenir football into the air from the top of the bleachers toward the bottom of the bleachers. The table shows the height of the football, in feet, above the ground at various times, in seconds. If the football wasn't touched by anyone on its way to the ground, about how long did it take the football to reach the ground after it was thrown?

Time (s)	0	0.2	0.4	0.6	0.8	1.0
Height (ft)	10	11.76	12.24	11.44	9.36	6.0

HABITS OF MIND

Model With Mathematics How many points does it take to determine the equation of a quadratic function? Why are so many more points used in Example 4?



Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION What key features can you determine about a quadratic function from an equation in standard form?

2. Error Analysis Cameron said that the y-intercept of a quadratic function always tells the maximum value of that function. Explain Cameron's error.

3. Vocabulary Write a quadratic function in standard form.

4. Make Sense and Persevere Why do you need at least three points to graph a quadratic function when not given an equation?

Do You KNOW HOW?

Find the vertex and *y*-intercept of the quadratic function.

5.
$$y = 3x^2 - 12x + 40$$

6.
$$y = -x^2 + 4x + 7$$

For 7 and 8, find the maximum or minimum of the parabola.

7.
$$y = -2x^2 - 16x + 20$$

8.
$$y = -x^2 + 12x - 15$$

9. Find the equation in standard form of the parabola that passes through the points (0, 6), (-3, 15), and (-6, 6).

10.
$$y = 3x^2 + 6x - 2$$

11.
$$y = -2x^2 + 4x + 1$$



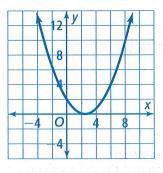
UNDERSTAND

12. Construct Arguments Devin found the parabola that fits the three points in the table to be $y = 0.345x^2 - 0.57x - 2.78$. Is Devin correct? Explain.

Х	-4	0.6	9
у	5	-3	20

13. Generalize How can you find the maximum or minimum value of a quadratic function?

14. Higher Order Thinking The quadratic function whose graph is shown represents a cereal bowl. Its equation is $y = 0.32x^2 - 1.6x + 2$. Describe how you could use the function to find the diameter of the cereal bowl if you know its depth.



15. Error Analysis Micah found the vertex for the function $y = -9.5x^2 - 47.5x + 63$ as shown.

$$x = -\frac{b}{2a}$$

$$x = -\frac{47.5}{2(-9.5)}$$

$$x = -\frac{47.5}{-19}$$

x = -(-2.5)

$$x = 2.5$$

$$y = -9.5(2.5)^2 - 47.5(2.5) + 63$$

$$y = -59.375 - 118.75 + 63$$

$$y = -115.125$$

Find and correct Micah's error.

PRACTICE

Find the vertex of each parabola. SEE EXAMPLE 1

16.
$$y = -x^2 + 6x + 30$$

17.
$$y = 3x^2 + 12x - 5$$

Find the vertex and y-intercept of the quadratic function, and use them to graph the function. SEE EXAMPLES 1 AND 2

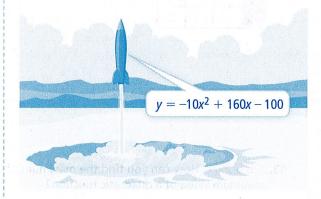
18.
$$y = -x^2 + 6x - 8$$

19.
$$y = x^2 - 8x + 11$$

20.
$$y = 3x^2 + 18x + 10$$

21.
$$y = -2x^2 - 12x - 5$$

22. A rocket is launched into the air. The path of the rocket is modeled by the equation $y = -10x^2 + 160x - 100$. What is the maximum height reached by the rocket, in feet? **SEE EXAMPLE 3**



Write the equation of a quadratic function in standard form for the parabola that passes through the given points. SEE EXAMPLE 4

Use quadratic regression to find the equation of a quadratic function that fits the given points.

SEE EXAMPLE 5

25.	X	0	0.5	1	1.5	2
	У	35	36	29	14	-9





APPLY

26. Model With Mathematics The height of Amelia's mid-section was measured three times during a long jump.

Time in seconds, <i>x</i>	0	0.5	1
Height in meters, y	0.7	1.5	0.55



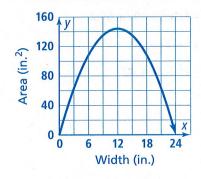
Write the equation of a quadratic function that describes Amelia's height as a function of time.

- 27. Make Sense and Persevere A college's business office found the relationship between the number of admissions counselors they employ and the college's profit from tuition could be modeled by the function $y = -10x^2 + 1,500x - 35,000.$
 - a. Graph the function.

b. How many admissions counselors should the college employ to maximize its profit?

c. What is the maximum amount of profit the college can make?

28. Mathematical Connections A rectangular tile has a perimeter of 48 inches.



a. The graph shows the relationship between the width of the tile and the area of the tile. What function describes this relationship?

b. What is the maximum area? What length and width give the maximum area?



ASSESSMENT PRACTICE

- **29.** Consider the quadratic function $y = 5x^2 50x 100$. Which of the following are true? Select all that apply.
 - A Its vertex is (5, -225).
 - $^{\textcircled{B}}$ Its *y*-intercept is (0, 100).
 - \bigcirc It passes through the point (-1, -45).
 - ① The minimum height occurs when x = 5.
 - **E** The maximum height occurs when x = 5.

Part B How high is the platform the diver jumped from? What is the maximum height reached?

Part A Find the equation of the quadratic

between the diver's time and height. Round to

function that describes the relationship

the nearest tenth.

30. SAT/ACT Which quadratic equation contains the three points (–4, 12), (2, 42), and (3, 40)?

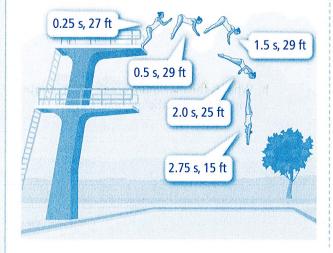
$$By = 1.7x^2 - 10x - 55.2$$

©
$$y = -1.7x^2 + 10x + 55.2$$

①
$$y = x^2 - 3x - 40$$

$$E y = -x^2 + 3x + 40$$

31. Performance Task A diver jumped from a diving platform. The image shows her height above the water at several different times after leaving the platform.



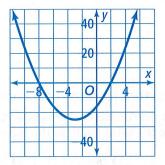
Part C From the maximum height, how long does it take the diver to get halfway down? Which part of the dive is faster, from the top to the halfway point, or from the halfway point to the water? Explain.



CRITIQUE & EXPLAIN

Corey wrote an equation in factored form, y = (x + 8)(x - 2), to represent a quadratic function. Kimberly wrote the equation $y = x^2 + 6x - 16$, and Joshua wrote the equation $y = (x + 3)^2 - 25$.

A. Reason Do all three equations represent the same function? If not, whose is different? Explain algebraically.



2-3
Factored Form of a Quadratic Function

B. How else could you determine if all three equations represent the same function?

C. What information can Corey's form help you find that is more difficult to find using Kimberly's or Joshua's form?

HABITS OF MIND

Use Structure Whose form of the equation is most useful for finding the vertex? The *y*-intercept? The *x*-intercepts?



Try It! Factor a Quadratic Expression **EXAMPLE 1**

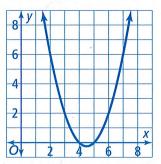
1. Factor the expression.

a.
$$x^2 - 9$$

b.
$$3x^2 - 7x + 2$$

Try It! Relate Factors to Zeros of a Function **EXAMPLE 2**

2. The graph shows the function $y = x^2 - 9x + 20$. Identify the zeros of the function. How do the zeros relate to the factors of $x^2 - 9x + 20$?



Try It! Solve Quadratic Equations by Factoring **EXAMPLE 3**

3. Solve the equation by factoring.

a.
$$x^2 + 8x = 20$$

b.
$$2x^2 = 3x + 2$$

HABITS OF MIND

Error Analysis Anna solved the equation $x^2 + 8x - 20 = 0$ by factoring. She wrote (x - 10)(x + 2) = 0 and expected the x-intercepts of the function $y = x^2 + 8x - 20$ to be at -10 and 2. Was she right?





EXAMPLE 4 Try It! Find the Zeros of a Quadratic Function

4. A baseball is thrown from the upper deck of a stadium, 128 ft above the ground. The function $h(x) = -16x^2 + 32x + 128$ gives the height of the ball x seconds after it is thrown. How long will it take the ball to reach the ground?

EXAMPLE 5



Try It! Determine Positive or Negative Intervals

5. Identify the interval(s) on which the function $y = x^2 - 4x - 21$ is negative.

HABITS OF MIND

Construct Arguments Is it always true that the y-values of a quadratic function have opposite signs on either side of a zero of the function? Explain why or give a counterexample.



Try It! Write the Equation of a Parabola in Factored Form

6. Write an equation of a parabola with x-intercepts at (3, 0) and (-3, 0) and which passes through the point (1, 2).

HABITS OF MIND

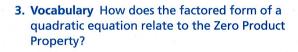
Model With Mathematics Is there any other parabola with x-intercepts at (-2, 0) and (-1, 0)? Give an equation or explain why there is no such parabola.



© Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How is the factored form helpful in solving quadratic equations?

2. Error Analysis Amir says the graph of $y = x^2 + 16$ has -4 as a zero. Is Amir correct? Explain.



4. Generalize How does knowing the zeros of a function help determine where a function is positive?

Do You KNOW HOW?

Factor each expression.

5.
$$x^2 - 5x - 24$$

6.
$$5x^2 + 3x - 2$$

Solve each equation.

7.
$$x^2 = 12x - 20$$

8.
$$4x^2 - 5x = 6$$

9. The height, in feet, of a t-shirt launched from a t-shirt cannon high in the stands at a football stadium is given by $h(x) = -16x^2 + 64x + 80$, where x is the time in seconds after the t-shirt is launched. How long will it take before the t-shirt reaches the ground?





UNDERSTAND

- **10. Generalize** Can you write the equation of a quadratic function knowing its zeros and its non-zero *y*-intercept? If so, describe the process. If not, explain why.
- **11. Error Analysis** Describe and correct the error a student made in solving a quadratic equation.

$$0 = 2x^{2} + 7x + 5$$

$$0 = 2x^{2} + 2x + 5x + 5$$

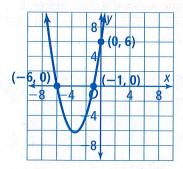
$$0 = 2x(x + 1) + 5(x + 1)$$

$$0 = 2x, 0 = x + 1, 0 \neq 5$$

$$0 = x, -1 = x$$



12. Model With Mathematics Use the graph of the function to write the equation in factored form.



13. Generalize For what values of x is the expression $(x-4)^2 > 0$?

14. Error Analysis A student says that the zeros of y = (x - 2)(x + 7) are -2 and 7. Is the student correct? If not, describe and correct the error the student made.

15. Construct Arguments Explain why $x^2 + 25$ is not equal to $(x + 5)^2$.

16. Mathematical Connections Describe how factoring can help you find the *x*-intercepts of the graph of the quadratic function $y = x^2 - 4x + 3$.



PRACTICE

Factor each quadratic expression. SEE EXAMPLE 1

17.
$$x^2 - 3x - 10$$

18.
$$3x^2 - 5x - 12$$

19.
$$x^2 + 15x + 56$$

20.
$$2x^2 + 7x - 15$$

21.
$$3x^2 - 18x - 48$$

22.
$$4x^2 - 11x - 3$$

23. What are the zeros of the quadratic function y = 3(x - 5)(x + 4)? SEE EXAMPLE 2

Solve each quadratic equation. SEE EXAMPLE 3

24.
$$x^2 - 5x - 14 = 0$$

25.
$$x^2 = 5x - 6$$

$$3x^2 - 60 = 3x$$

26.
$$3x^2 - 60 = 3x$$
 27. $5x^2 + 12x = 9$

28.
$$4x^2 + 3x - 7 = 0$$

29.
$$6x^2 = 5x + 6$$

30. A penny is dropped from the top of a new building. Its height in feet can be modeled by the equation $y = 256 - 16x^2$, where x is the time in seconds since the penny was dropped. How long does it take for the penny to reach the ground? SEE EXAMPLE 4

Identify the interval(s) on which each quadratic function is positive. SEE EXAMPLE 5

31.
$$y = x^2 + 9x + 18$$

32.
$$y = x^2 + 2x - 8$$

33.
$$y = x^2 - 5x - 24$$

33.
$$y = x^2 - 5x - 24$$
 34. $y = -x^2 + 4x + 12$

35.
$$y = 2x^2 + 12x + 18$$
 36. $y = 5x^2 - 3x - 8$

36.
$$y = 5x^2 - 3x - 8$$

Write an equation for each parabola. SEE EXAMPLE 6

37. A parabola with x-intercepts at (-1, 0) and (3, 0) which passes through the point (1, -8)

38. A parabola with x-intercepts at 0 and 1 and which passes through the point (2, -2)

39. A snorkeler dives for a shell on a reef. After entering the water, the diver descends $\frac{11}{3}$ ft in one second. Write an equation that models the diver's position with respect to time.

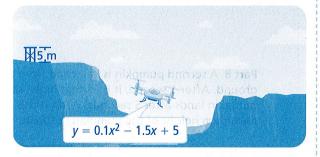




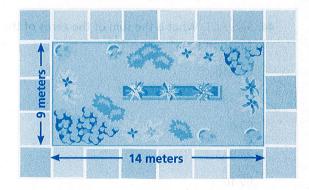
APPLY

- **40.** Make Sense and Persevere Rectangular apartments are 12 ft longer than they are wide. Each apartment has 1,053 ft² of floor space. What are the dimensions of an apartment? Explain.
- c. After how many seconds will the drone come back to the height of its platform?

41. Use Structure The height of a drone, in meters, above its launching platform that is 5 m above the ground, is modeled by $y = 0.1x^2 - 1.5x + 5$, where x is the time in seconds. The drone leaves the launch pad, flies down into a canyon, and then it flies back up again.



42. Higher Order Thinking LaTanya is designing a rectangular garden with a uniform walkway around its border. LaTanya has 140 m² of material to build the walkway.



- a. What is the factored form of the equation for the height of the drone?
- a. Write an equation for the dimensions of the garden and the surrounding walkway.

- **b.** After how many seconds will the drone be at ground level?
- **b.** How wide is the walkway? Explain.

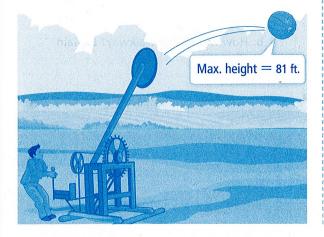


ASSESSMENT PRACTICE

- **43.** Which of the following are solutions to the equation $-11x = 2x^2 + 15$? Select all that apply.
 - \bigcirc -5
 - ® −3
 - $\bigcirc -\frac{5}{2}$
 - (D)
 - **E** 3
 - **E** 5

- **44. SAT/ACT** What is the sum of the zeros of the function $y = x^2 9x 10$?
 - A -10
 - B -9
 - © 0
 - **D** 9
 - **E** 10

45. Performance Task A pumpkin is launched from the ground into the air and lands 4.5 s later.



Part A Write a quadratic function that models the height, in feet, of the pumpkin *x* seconds after it is launched. Explain how you found the function.

Part B A second pumpkin is launched from the ground. After 1 second, it is 64 feet high. The pumpkin lands after 5 seconds. What is the maximum height of the pumpkin? Explain.



EXPLORE & REASON

A math class played a game called "Solve It, You're Out." At the start of each round, students chose a card from a deck marked with integers from -5 to 5. When an equation is shown, any student whose card states the solution to the equation is eliminated. Five students remain.

Five remaining students with the cards they hold:

Mercedes: -3

Steve: 0

Aubrey: 1

Elijah: -2

Fatima: 3



A. The next equation presented was $x^2 = 9$. Which student(s) was eliminated? Explain.

B. Construct Arguments In the next round, the equation presented was $x^2 = -4$. Darius thought he was eliminated, but this is not the case. Explain why Darius was incorrect.

C. What is true about solutions to $x^2 = a$ when a is a positive number? When a is a negative number? What about when a = 0?

HABITS OF MIND

Reason Steve thought that he was a sure winner because he could not be eliminated. Is he correct? Explain. If not, write an equation of the form $x^2 = a$ that would eliminate Steve.



EXAMPLE 1



Try It! Solve a Quadratic Equation Using Square Roots

1. Use square roots to solve each equation. Write your solutions using the imaginary unit, i.

a.
$$x^2 = -5$$

b.
$$x^2 = -72$$

HABITS OF MIND

Communicate Precisely How do you know that the solution to the equation $x^2 = -5$ must be an imaginary number?



Try It! Add and Subtract Complex Numbers

2. Find the sum or difference.

a.
$$(-4+6i)+(-2-9i)$$

b.
$$(3-2i)-(-4+i)$$

HABITS OF MIND

Generalize How is adding and subtracting complex numbers similar to adding and subtracting binomials?



Try It! Multiply Complex Numbers

3. Write each product in the form a + bi.

a.
$$\frac{2}{5}i\left(10 - \frac{5}{2}i\right)$$

b.
$$\left(\frac{1}{2} + 2i\right) \left(\frac{1}{2} - 2i\right)$$

Try It! Simplify a Quotient With Complex Numbers

4. Write each quotient in the form a + bi.

a.
$$\frac{80}{2-6i}$$

b.
$$\frac{4-3i}{-1+2i}$$

HABITS OF MIND

Use Structure Why do you multiply the numerator and denominator of a complex fraction by the conjugate of the denominator?

Try It! Factor a Sum of Squares

5. Factor each expression.

a.
$$4x^2 + 25$$

b.
$$8y^2 + 18$$

Try It! Solve a Quadratic Equation With Complex Solutions **EXAMPLE 6**

6. Find the value(s) of x that will solve each equation.

a.
$$x^2 + 49 = 0$$

b.
$$9x^2 + 25 = 0$$

HABITS OF MIND

Construct Arguments For what values of a will the solutions to the equation $x^2 - a = 0$ be complex numbers? Explain how you know.





Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How can you represent and operate on numbers that are not on the real number line?

2. Vocabulary How do you form the complex conjugate of a complex number a + bi?

3. Error Analysis Helena was asked to write the quotient $\frac{4}{3-i}$ in the form a+bi. She began this way: $\frac{4}{3-i} \times \frac{3-i}{3-i} = \frac{4(3-i)}{3^2+1^2} = \frac{12-4i}{10}$. Explain

4. Look for Relationships The quadratic equation $x^2 + 9 = 0$ has solutions x = 3i and x = -3i. How many times will the graph of $f(x) = x^2 + 9$ cross the x-axis? Explain.

Do You KNOW HOW?

Write each of the following in the form a + bi.

5.
$$(2+5i)-(-6+i)$$

6.
$$(2i)(6+3i)$$

Solve each equation.

7.
$$x^2 + 16 = 0$$

8.
$$y^2 = -25$$

9. The total source voltage in the circuit is 6 - 3i volts. What is the voltage at the middle source?

$$(2+6i)V \bigcirc_{-}^{+} E_{1}$$

$$(a+bi)V \bigcirc_{-}^{+} E_{2}$$

$$(2-5i)V \bigcirc_{-}^{+} E_{3}$$





UNDERSTAND

10. Construct Arguments Tamara says that raising the number i to any integer power results in either -1 or 1 as the result, since $i^2 = -1$. Do you agree with Tamara? Explain.

11. Error Analysis Describe and correct the error a student made when dividing complex numbers.

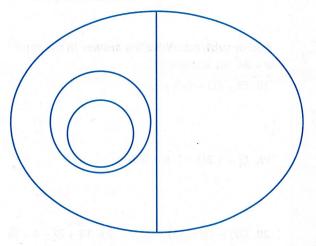
$$\frac{1+i}{3-i} =$$

$$\frac{1+i}{3-i} \cdot \frac{1-i}{3+i} =$$

$$\frac{1-i^2}{9-i^2} =$$

$$\frac{2}{10}$$

- 12. Higher Order Thinking Label the diagram with the following sets of numbers:
 - 1. complex numbers
 - 2. real numbers
 - 3. imaginary numbers
 - 4. integers
 - 5. rational numbers



Include an example of each type of number in the diagram.

13. Generalize Write an explicit formula, in standard form, to find the quotient of two complex numbers. Use the numbers a + biand c + di.



PRACTICE

Use square roots to solve each equation over the complex numbers. SEE EXAMPLE 1

14.
$$x^2 = -5$$

15.
$$x^2 = -0.01$$

16.
$$x^2 = -18$$

17.
$$x^2 = (-1)^2$$

Add or subtract. Write the answer in the form a + bi. SEE EXAMPLE 2

18.
$$(3-2i)-(-9+i)$$

19.
$$(5 + 1.2i) + (-6 + 0.8i)$$

20.
$$(2i) - (2i - 11)$$

21.
$$13 + 2i - 4 - 8i$$

22.
$$\frac{3-i}{4} - \frac{2+i}{3}$$

23.
$$4.5i - 4.5 + 3.5i + 2.5$$

Write each product in the form a + bi. SEE EXAMPLE 3

25.
$$(3i)(5-4i)$$

26.
$$(5-2i)(5+2i)$$

27.
$$(8 + 3i)(8 + 3i)$$

28.
$$\frac{1}{3}i(3+6i)$$

29.
$$(-2i+7)(7+2i)$$

Write each quotient in the form a + bi. SEE EXAMPLE 4

30.
$$\frac{12}{1-i}$$

31.
$$\frac{5}{6+2i}$$

32.
$$\frac{6+12i}{3i}$$

33.
$$\frac{4-4i}{1+3i}$$

Factor the sums of two squares. SEE EXAMPLE 5

34.
$$4x^2 + 49$$

35.
$$x^2 + 1$$

36.
$$36 + 100a^2$$

37.
$$18y^2 + 8$$

38.
$$\frac{1}{4}b^2 + 25$$

39.
$$x^2 + y^2$$

Solve each equation. SEE EXAMPLE 6

40.
$$x^2 + 81 = 0$$

40.
$$x^2 + 81 = 0$$
 41. $25x^2 + 9 = 0$

42.
$$x^2 = -16$$

43.
$$4 + 49y^2 = 0$$

$$44 v^2 + 1 - 6$$

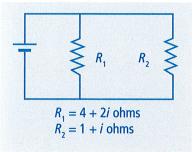
44.
$$y^2 + 1 = 0$$
 45. $x^2 + \frac{1}{4} = 0$



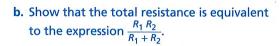


APPLY

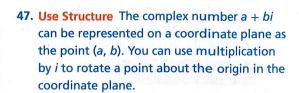
46. Model With Mathematics The two resistors shown in the circuit are referred to as *in parallel*. The total resistance of the resistors is given by the formula $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$.

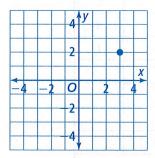


a. Find the total resistance. Write your answer in the form a + bi.



c. Change the value of R_2 so that the total resistance is a real number. Explain how you chose the value.





a. Write the point (x, y) on the graph as the complex number x + yi.

b. Multiply the complex number by *i*. Interpret the new value as a new point in the plane.

c. Repeat the steps above for two other points. How does multiplication by i rotate a point?



ASSESSMENT PRACTICE

48. Complete the table by classifying each number as real, imaginary, or complex. Use the most specific classification. For example, all real numbers are also complex numbers, so it is more specific to classify a number as real.

Number	R, I, C
2 + <i>i</i>	С
5 — 0 <i>i</i>	R
- 2i	1
$(3-i)^2$	
$i^2 + 1$	
3 <i>i</i>	
(3-i)(3+i)	
(3+i)-(2+i)	1
$\sqrt{-14}$	
i(4+i)-3i	

- 49. **SAT/ACT** Which of the following is a solution to the equation $3x^2 = -12$?
 - \bigcirc -4i
 - B −2*i*
 - © −2
 - **D** 2
 - **E** 4*i*
- **50. Performance Task** Abby wants to write the square root of *i* in the form a + bi. She begins by writing the equation $\sqrt{i} = a + bi$.

Part A Square both sides of the equation. Then use the fact that the real part and imaginary part on each side of the equation are equal to write a system of equations involving the variables a and b.

Part B Solve the system to find b. Then find a.

Part C List the possible solutions for a and b.

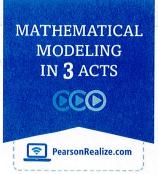
Part D Square each of the possible solutions. What are the two square roots of *i*?



Swift Kick

Whether you call it soccer, football, or fùtbol, it's the most popular sport in the world by far. Even if you don't play soccer, you probably know several people who do.

There are many ways to kick a soccer ball: you can use any part of either foot. If you want the ball to end up in the goal, you also need to try different amounts of spin and power. You'll see one person's effort in the Mathematical Modeling in 3-Acts lesson.



ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?

2. Write down the Main Question you will answer.

3. Make an initial conjecture that answers this Main Question.

4. Explain how you arrived at your conjecture.

5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?



Develop a Model

6. Use the math that you have learned in the topic to refine your conjecture.

Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

PearsonRealize.com

CRITIQUE & EXPLAIN

Hana and Enrique used different methods to solve the equation $x^2 - 6x + 9 = 16$.

Hana

$$x^2 - 6x + 9 = 16$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1)=0$$

$$x - 7 = 0$$
 OR $x + 1 = 0$

$$x = 7$$
 OR

$$x = -1$$

The solutions are 7 and -1.

Enrique

$$x^2 - 6x + 9 = 16$$

$$(x-3)^2=16$$

I can square 4 or -4 to get 16.

$$x - 3 = 4$$
 OR $x - 3 = -4$.

$$x = 7$$
 OR

$$x = -1$$

The solutions are 7 and -1.

A. Does Hana's method work? If her method is valid, explain the reasoning she used. If her method is not valid, explain why not.

B. Does Enrique's method work? If his method is valid, explain the reasoning he used. If his method is not valid, explain why not.

C. Use Structure Can you use either Hana's or Enrique's method to solve the equation $x^2 + 10x + 25 = 3$? Explain.

HABITS OF MIND

Make Sense and Persevere Why does Hana set her two factors equal to zero, while Enrique sets his factor equal to 4 and -4?



EXAMPLE 1



Try It! Use Square Roots to Solve Quadratic Equations

1. Find the solution(s) to the equations.

a.
$$81 = x^2 + 12x + 36$$
?

b.
$$9 = x^2 - 16x + 64$$

HABITS OF MIND

Use Structure How do you recognize a perfect square trinomial?

EXAMPLE 2



Try It! Understand the Process of Completing the Square

2. How can you write the equation $x^2 - 6x - 11 = 0$ in the form $(x - p)^2 = q$?

EXAMPLE 3



Try It! Solve a Quadratic Equation by Completing the Square

3. Solve the following equations by completing the square.

a.
$$0 = x^2 + 4x + 8$$

b.
$$0 = x^2 - 8x + 17$$

HABITS OF MIND

Reason Richard is completing the square to solve the equation $2x^2 + 8x = 19$. He wrote: $2(x^2 + 4x + 4) = 19 +$ ___. What number should Richard write in the blank?



Try It! Complete the Square to Solve a Real-World Problem **EXAMPLE 4**

4. The relationship between the time since the ball was thrown and height of the ball can be modeled by the equation $h = 32t - 16t^2 + 4$, where h is the height of the ball after t seconds. Complete the square to find how long it will take the ball to reach a height of 20 ft.

Try It! Write a Quadratic Equation in Vertex Form **EXAMPLE 5**

5. Write each equation in vertex form. Identify the maximum or minimum value of the graph of each equation.

a.
$$y = -3x^2 - 9x + 7$$

b.
$$y = 2x^2 + 12x + 9$$

HABITS OF MIND

Make Sense and Persevere A pelican swoops down under the surface of the ocean to catch a fish. An equation that describes the pelican's path is $y = 4x^2 - 16x + 15$. The pelican catches the fish at the deepest point of the dive. How deep was the fish swimming?



Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How can you solve a quadratic equation by completing the square?

2. Paula said that only quadratic equations with leading coefficients of 1 can be solved by completing the square. Is Paula correct? Explain.

3. Generalize Given the expression $x^2 + bx$, describe how to find c so that $x^2 + bx + c$ is a perfect square trinomial.

4. Make Sense and Persevere How can you complete the square to find the vertex of a parabola?

Do You KNOW HOW?

Solve each equation by completing the square.

5.
$$0 = x^2 + 12x + 11$$

6.
$$27 = 3x^2 + 12x$$

7.
$$0 = 2x^2 + 6x - 14$$

Write the equation in vertex form, and identify the maximum or minimum point of the graph of the function.

8.
$$y = x^2 + 6x - 6$$

9.
$$y = -2x^2 + 20x - 42$$

10. The daily profit for a company is modeled by the function $p(x) = -0.5x^2 + 40x - 300$, where x is the number of units sold. How many units does the company need to sell each day to maximize profits?

UNDERSTAND

- 11. Use Appropriate Tools How could you use a graphing calculator to determine whether you have correctly solved a quadratic equation by completing the square?
- 14. Reason Does the geometric model hold for finding the number that completes the square of the expression $x^2 - 12x$? Explain.

12. Error Analysis Describe and correct the error a student made in solving a quadratic equation by completing the square.

$$0 = x^{2} + 16x - 5$$

$$5 = x^{2} + 16x + 64$$

$$5 = (x + 8)^{2}$$

$$x = -8 \pm \sqrt{5}$$

15. Error Analysis When given the equation $-23 = x^2 + 8x$, a student says that you can add 64 to each side of the equation to complete the square. Is the student correct? If not, describe and correct the error.

16. Construct Arguments Explain why you should not try to complete the square when solving $0 = x^2 - 4$.

- 13. Higher Order Thinking What number do you need to add to $x^2 + \frac{7}{2}x$ in order to create a perfect square trinomial? Explain.
- 17. Use Structure Jacob completed the square to rewrite the equation $f(x) = -2x^2 + 12x - 13$ as $f(x) = -2(x - 3)^2 + 5$. Which form of the equation is more helpful for identifying the key features of the graph? Explain.



PRACTICE

Use square roots to solve the quadratic equations. **SEE EXAMPLE 1**

18.
$$9 = x^2 + 2x + 1$$

19.
$$16 = x^2 - 10x + 25$$

20.
$$50 = 2x^2 + 16x + 32$$

21.
$$5 = 3x^2 - 36x + 108$$

22.
$$7 = x^2 + 4x + 4$$

23.
$$-4 = x^2 + 14x + 49$$

Rewrite the equations in the form $(x - p)^2 = q$. SEE EXAMPLE 2

24.
$$0 = x^2 - 18x + 64$$

25.
$$x^2 + 22x + 120.5 = 0$$

26.
$$x^2 + 3x - \frac{27}{4} = 0$$
 27. $0 = 4x^2 + 4x - 14$

27.
$$0 = 4x^2 + 4x - 14$$

28.
$$0 = x^2 - \frac{3}{2}x - \frac{70}{8}$$

29.
$$x^2 + 0.6x - 19.1 = 0$$

Solve the following quadratic equations by completing the square. SEE EXAMPLES 3 AND 4

30.
$$x^2 + 8x + 60 = 0$$
 31. $x^2 + 14x = 51$

31
$$x^2 + 14x = 5$$

32
$$4v^2 + 16v - 65 - 6$$

32.
$$4x^2 + 16x - 65 = 0$$
 33. $7x^2 + 56x - 22 = 0$

34.
$$3x^2 - 6x + 13 = 0$$

35.
$$x^2 - 0.4x - 1.2 = 0$$

36.
$$x^2 + 6x = 59$$

37.
$$8x^2 + 16x = 42$$

38.
$$5x^2 - 25 = 10x$$

39.
$$-2x^2 - 12x + 18 = 0$$

40.
$$-3x^2 - 24x - 19 = 0$$
 41. $17 - x^2 - 18x = 0$

41.
$$17 - x^2 - 18x = 0$$

42. What is the length and width of the skate park?



Write the equation in vertex form. Identify the maximum or minimum value of the graph of the equation. SEE EXAMPLE 5

43.
$$y = x^2 + 4x - 13$$
 44. $y = x^2 - 14x + 71$

44.
$$y = x^2 - 14x + 71$$

45.
$$y = -2x^2 - 20x - 58$$

46.
$$y = -3x^2 + 36x - 93$$

47.
$$v = 6x^2 - 42x + 74.5$$

48.
$$y = 0.5x^2 + 0.5x + 2.125$$





APPLY

49. Make Sense and Persevere Keenan launches a model helicopter. The height of the helicopter, in feet, is given by the equation $h = -16t^2 + 64t + 190$, where t is the time in seconds. To the nearest hundredth, how many seconds will it take the helicopter to hit the ground? What is the maximum height of the helicopter?



- **50.** Use Structure The decreasing population, p, of owls in a national park is being monitored by ecologists and is modeled by the equation $p = -0.4t^2 + 128t + 1,200$, where t is the number of months since the ecologists started observing the owls.
 - a. If this model is accurate, when will the population reach its maximum?

b. What is the maximum population? Round to the nearest whole number.

c. Use the equation to determine in how many months the population of owls will disappear.

51. Make Sense and Persevere Between 2000 and 2005, the number of skateboarders s in the United States, in millions, can be approximated by the equation $s = 0.33t^2 + 2.27t + 3.96$, where t represents the number of years since 2000. If this model is accurate, in what year did 9.8 million people skateboard?



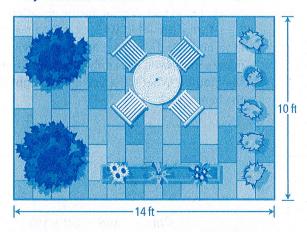
ASSESSMENT PRACTICE

52. The roots of $f(x) = -2x^2 + 8x + 13$ are

 $_{\rm and}$

The vertex of the parabola is at

54. Performance Task Yumiko has a rectangular-shaped patio. She wants to double the area of the patio by increasing the length and width by the same amount.



Part A Write a function to calculate the number of feet Yumiko would need to add to the length and width. Explain your reasoning.

53. SAT/ACT Solve $x^2 + 2x - 5 = 0$.

$$\triangle$$
 -5, 1

$$^{\odot}$$
 −1 ± $\sqrt{5}$

©
$$-1 \pm \sqrt{6}$$

①
$$1 \pm \sqrt{5}$$

E −3, 1

Part B To the nearest hundredth, what are the new dimensions of the patio?

EXPLORE & REASON

You can complete the square to solve the general quadratic equation, $ax^2 + bx + c = 0.$

$$ax^{2} + bx + c = 0$$

$$ax^{2} + bx = -c$$

$$x^{2} + \left(\frac{b}{a}\right)x = -\frac{c}{a}$$

$$x^{2} + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^{2} = -\frac{c}{a} + \left(\frac{b}{2a}\right)^{2}$$

$$\left(x + \frac{b}{2a}\right)^{2} = \frac{b^{2}}{4a^{2}} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^{2} = \frac{b^{2} - 4ac}{4a^{2}}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^{2} - 4ac}{4a^{2}}}$$





A. Construct Arguments Justify each step in this general solution.

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

B. What must be true of the value of $b^2 - 4ac$ if the equation $ax^2 + bx + c = 0$ has two non-real solutions? If it has just one solution?

HABITS OF MIND

Communicate Precisely Why is there a \pm in the second to last step of the derivation of the Quadratic Formula?



- 1. Solve using the Quadratic Formula.
 - a. $2x^2 + 6x + 3 = 0$
- **b.** $3x^2 2x + 7 = 0$

EXAMPLE 2 Try It! Choose a Solution Method

2. Solve the equation $6x^2 + x - 15 = 0$ using the Quadratic Formula and one other method.

HABITS OF MIND

Construct Arguments Is it possible for a quadratic equation to have one real solution and one complex solution? Explain.

EXAMPLE 3 Try It! Identify the Number of Real-Number Solutions

- 3. Describe the nature of the solutions for each equation.
 - a. $16x^2 + 8x + 1 = 0$
- **b.** $2x^2 5x + 6 = 0$





EXAMPLE 4 Try It! Interpret the Discriminant

4. According to the model of Rachel's serve, will the ball reach a height of 3 meters?

HABITS OF MIND

Reason Create a quadratic equation that has two complex solutions.



5. Determine the value(s) of b that ensure $5x^2 + bx + 5 = 0$ has two non-real solutions.

HABITS OF MIND

Use Appropriate Tools Why is the Quadratic Formula helpful?



Do You UNDERSTAND?

1.? ESSENTIAL QUESTION How can you use the Quadratic Formula to solve quadratic equations or to predict the nature of their solutions?

2. Vocabulary Why is the discriminant a useful tool to use when solving quadratic equations?

3. Error Analysis Rick claims that the equation $x^2 + 5x + 9 = 0$ has no solution. Jenny claims that there are two solutions. Explain how Rick could be correct, and explain how Jenny could be correct.

4. Use Appropriate Tools What methods can you use to solve quadratic equations?

Do You KNOW HOW?

5. Describe the number and type of solutions of the equation $2x^2 + 7x + 11 = 0$.

6. Use the Quadratic Formula to solve the equation $x^2 + 6x - 10 = 0$.

7. At time t seconds, the height, h, of a ball thrown vertically upward is modeled by the equation $h = -5t^2 + 33t + 4$. About how long will it take for the ball to hit the ground?

8. Use the Quadratic Formula to solve the equation $x^2 - 8x + 16 = 0$. Is this the only way to solve this equation? Explain.





UNDERSTAND

9. Look for Relationships How can you use the Quadratic Formula to factor a quadratic equation?

10. Error Analysis Describe and correct the error a student made in solving an equation.

$$x^{2}-5x+5=0$$

$$a = 1, b = -5, c = 5$$

$$x = \frac{-5 \pm \sqrt{(-5)^{2}-4(1)(5)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25-20}}{2}$$

$$= \frac{-5}{2} \pm \frac{\sqrt{5}}{2}$$

11. Mathematical Connections What does the Quadratic Formula tell you about the graph of a quadratic function?

12. Communicate Precisely Explain your process for choosing a method for solving quadratic equations.

13. Higher Order Thinking Kelsey wants to use the Quadratic Formula to solve the equation $x^4 + 5x^2 - 5 = 0$. Is this possible? If so, describe the steps she should follow.

14. Construct Arguments Explain why the graph of the quadratic function $f(x) = x^2 + x + 5$ crosses the y-axis but does not cross the x-axis.

15. Construct Arguments Sage said that the Quadratic Formula does not always work. Sage used it to solve the equation $x^2 - 3x - 2 = -4$, with a = 1, b = -3, and c=-2. The formula gave $x=\frac{3\pm\sqrt{17}}{2}$ as the solutions to the equation. When Sage checked, neither one of them satisfied the equation. How could you convince Sage that the Quadratic Formula does always work?



PRACTICE

Use the Quadratic Formula to solve each equation. SEE EXAMPLE 1

16.
$$x^2 - 10x + 25 = 0$$
 17. $x^2 + 2x + 2 = 0$

17.
$$x^2 + 2x + 2 = 0$$

18.
$$5x^2 - 8x + 4 = 0$$

18.
$$5x^2 - 8x + 4 = 0$$
 19. $x^2 + 9x - 1 = 3x - 10$

20.
$$3x^2 - 20x - 7 = 0$$

21.
$$-x^2 + 3x - 8 = 0$$

Use the discriminant to identify the number and type of solutions for each equation. SEE EXAMPLE 3

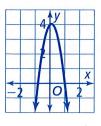
22.
$$25x^2 - 20x + 4 = 0$$
 23. $x^2 + 7x + 11 = 0$

23.
$$x^2 + 7x + 11 = 0$$

24.
$$3x^2 - 8x - 10 = 0$$
 25. $2x^2 + 9x + 14 = 0$

25.
$$2x^2 + 9x + 14 = 0$$

Deon throws a ball into the air. The height, h, of the ball, in meters, at time t seconds is modeled by the function $h(t) = -5t^2 + t + 4$. SEE EXAMPLE 4



26. When will the ball hit the ground?

27. Will the ball reach a height of 5 meters?

Use any method to solve the equation. SEE EXAMPLE 2

28.
$$4x^2 + 7x - 11 = 0$$
 29. $x^2 + 4x + 4 = 100$

29.
$$x^2 + 4x + 4 = 100$$

30.
$$3x^2 + x + 7 = x^2 + 10$$
 31. $6x^2 + 2x + 3 = 0$

Find the value(s) of k that will cause the equation to have the given number and type of solutions. SEE EXAMPLE 5

32.
$$5x^2 + kx + 5 = 0$$
, 1 real solution

33.
$$3x^2 + 12x + k = 0$$
, 2 real solutions

34.
$$kx^2 - 3x + 4 = 0$$
, 2 real solutions



APPLY

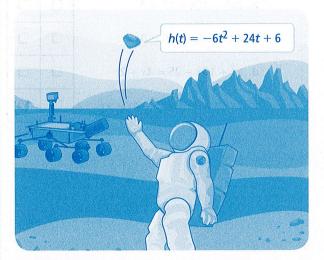
35. Model With Mathematics The table shows the average cost of tuition and fees at a public four-year college for an in-state student in recent years.

Academic Year	Tuition and Fees			
2012–13	\$9,006			
2013–14	\$9,077			
2014–15	\$9,161			
2015–16	\$9,410			

a. Write an equation that can be used to find the average cost, C, of tuition after x years.

b. Use the model to predict when tuition will exceed \$10,000.

36. Make Sense and Persevere The first astronaut on Mars tosses a rock straight up. The height, h, measured in feet after t seconds, is given by the function $h(t) = -6t^2 + 24t + 6$.



- a. After how many seconds will the rock be 30 feet above the surface?
- b. After how many seconds will the rock be 10 feet above the surface?
- c. How many seconds will it take for the rock to return to the surface?
- d. The same action on Earth is modeled by the equation $g(t) = -16t^2 + 24t + 6$. On Earth, how many seconds would it take for the rock to hit the ground?



ASSESSMENT PRACTICE

37. Which of the following equations has two real solutions? Select Yes or No.

	_						
a.	x^2	_	8x	_	2	=	0

b.
$$2x^2 + 10x + 17 = 0$$

c.
$$4x^2 - 28x + 49 = 0$$

d.
$$x^2 + 10x - 25 = 4x + 2$$

e.
$$2x^2 + x + 10 = 5 - 4x - x^2$$

Yes	No		

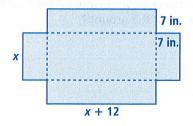
38. SAT/ACT Which expression can be simplified to find the solution(s) of the equation

$$2x^2 - x - 15 = 0$$
?

$$\bigcirc$$
 -1 $\pm \frac{\sqrt{1-4(2)(-15)}}{2(2)}$

©
$$\frac{1 \pm \sqrt{-1 - 4(2)(-15)}}{2(2)}$$

39. Performance Task Four congruent squares are cut from a rectangular piece of cardboard.



Part A. If the resulting flaps are folded up and taped together to make a box, write a function to represent the volume of the box in terms of the width of the original piece of cardboard.

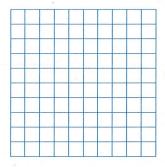
Part B. What are the dimensions of the original cardboard, to the nearest tenth, if the volume of the box is 434 in.³?



EXPLORE & REASON

Draw a rough sketch of a parabola and a line on the coordinate plane.

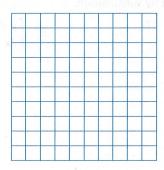
A. Count the number of points of intersection between the two graphs.



2-7 Linear-Quadratic **Systems**

PearsonRealize.com

B. Sketch another parabola on a coordinate plane. Use a straightedge to investigate the different ways that a line and a parabola intersect. What conjectures can you make?



C. Construct Arguments How many different numbers of intersection points are possible between a quadratic function and a linear function? Justify that you have found all of the possibilities.

HABITS OF MIND

Reason What must be true about the equation for a horizontal line that has no points of intersection with the parabola with equation $y = x^2$?





1. Determine the number of real solutions of the

system
$$\begin{cases} y = 3x^2 \\ y = 3x - 2 \end{cases}$$

EXAMPLE 2



Try It! Solve a Linear-Quadratic System Using Substitution

2. Solve each system by substitution.

a.
$$\begin{cases} y = 2x^2 - 6x - 8 \\ 2x - y = 16 \end{cases}$$

b.
$$\begin{cases} y = -3x^2 + x + 4 \\ 4x - y = 2 \end{cases}$$

EXAMPLE 3



Try It! Applying a Linear-Quadratic System

3. Revenue for the high school band concert is given by the function $y = -30x^2 + 250x$, where x is the ticket price, in dollars. The cost of the concert is given by the function y = 490 - 30x. At what ticket price will the band make enough revenue to cover their costs?

HABITS OF MIND

Make Sense and Persevere Why does the substitution method work? How does it change the problem and make it possible for you to solve?



Try It! Solve a Linear-Quadratic System of Inequalities **EXAMPLE 4**

4. Solve the system of inequalities $\begin{cases} y > x^2 + 6x - 12 \end{cases}$ using shading.

Try It! Using a System to Solve an Equation **EXAMPLE 5**

5. Solve the equation $3x^2 - 7x + 4 = 9 - 2x$ by writing a linear-quadratic system and solving using the intersection feature of a graphing calculator.

HABITS OF MIND

Look for Relationships How could you solve the inequality $3x + 8 > x^2 + 6x - 2$ graphically?



Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How can you solve a system of two equations or inequalities in which one is linear and one is quadratic?

4. $\begin{cases} y = -x - 1 \\ 3x^2 + 2y = 4 \end{cases}$

of equations.

3. $\begin{cases} y = \frac{2}{5}x^2 \\ y = x - 2 \end{cases}$

Do You KNOW HOW?

Determine the number of solutions for the system

2. Error Analysis Dyani was asked to use substitution to solve this system:

$$\begin{cases} y = 2x^2 - 6x + 4 \\ x - y = 7 \end{cases}$$

She began as follows, to find the *x*-coordinate(s) to the solution(s) of the system:

$$x + 2x^2 - 6x + 4 = 7$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3)=0$$

$$x = -\frac{1}{2}$$
, $x = 3$

Substitute for y.

Simplify.

Factor.

Set each factor equal to 0, solve for *x*.



But Dyani has already made an error. What was her mistake?

Use substitution to solve the system of equations.

5.
$$\begin{cases} y = 3x^2 + 7x - 10 \\ y - 19x = 22 \end{cases}$$

6.
$$\begin{cases} y = 3x^2 \\ y - 3x = -2 \end{cases}$$



UNDERSTAND

7. Construct Arguments Nora and William are asked to solve the system of equations (y - 1 = 3x)without graphing.

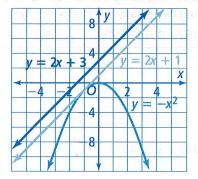
Nora wants to use substitution, inserting $2x^2 - 4x + 9$ in place of y in the upper equation and solving. William wants to rewrite y - 1 = 3x as y = 3x + 1 and begin by setting 3x + 1 equal to $2x^2 - 4x + 9$, and then solving. Which student is correct, and why?

of equations: $\begin{cases} y = x^2 \\ y = -1 \end{cases}$. Without graphing or performing any substitutions, can you see how many solutions the system must have? Describe your reasoning.

9. Reason You are given the following system

8. Error Analysis Chris was given the system of equations $\begin{cases} y = -x^2 \\ y = 2x + b \end{cases}$ and asked to use

graphing to test the number of solutions of the system for different values of b. He graphed the system as shown, and concluded that the system could have one solution or no solutions depending on the value of b. What was Chris's error?



10. Construct Arguments Can a system of equations with one linear and one quadratic equation have more than two solutions? Give at least two arguments for your answer.





PRACTICE

Determine how many solutions each system of equations has by graphing them. SEE EXAMPLE 1

11.
$$\begin{cases} y = 3 \\ y = x^2 - 4x + 7 \end{cases}$$

12.
$$\begin{cases} y = 3x^2 - 2x + 7 \\ y + 5 = \frac{1}{2}x \end{cases}$$

Consider the system of equations $\begin{cases} y = x^2 \\ y = mx + b \end{cases}$ **SEE EXAMPLE 1**

- **13.** Find values for *m* and *b* so that the system has two solutions.
- 14. Find values for m and b so that the system has no solutions.
- **15.** Find values for *m* and *b* so that the system has one solution.

Use substitution to solve the system of equations. **SEE EXAMPLE 2**

16.
$$\begin{cases} y = 5 \\ y = 2x^2 - 16x + 29 \end{cases}$$

17.
$$\begin{cases} y = 3x^2 - 4x \\ 27 + y = 14x \end{cases}$$

18. LaToya throws a ball from the top of a bridge. Her throw is modeled by the equation $y = -0.5x^2 + 3x + 10$, and the bridge is modeled by the equation y = -0.2x + 7. About how far does the ball travel horizontally before its first bounce? SEE EXAMPLE 3

Solve each system of inequalities using shading. **SEE EXAMPLE 4**

$$19. \begin{cases} y > x^2 \\ 5 > y \end{cases}$$

$$20. \begin{cases} -5 < y - x \\ y < -3x^2 + 6x + 1 \end{cases}$$

Solve each equation by writing a linear-quadratic system and solving using the intersection feature of a graphing calculator. SEE EXAMPLE 5

21.
$$6x^2 - 15x + 8 = 17 - 4x$$

22.
$$7x^2 - 28x + 32 = 4$$

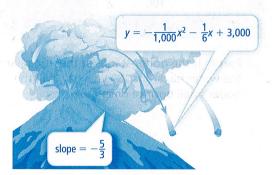
23.
$$-\frac{5}{2}x - 10 = -2x^2 - x - 3$$





APPLY

24. Model With Mathematics A boulder is flung out of the top of a 3,000 m tall volcano. The boulder's height, y, in meters, is a function of the horizontal distance it travels, x, in meters. The slope of the line representing the volcano's hillside is $-\frac{5}{3}$. At what height above the ground will the boulder strike the hillside? How far will it have traveled horizontally when it crashes?

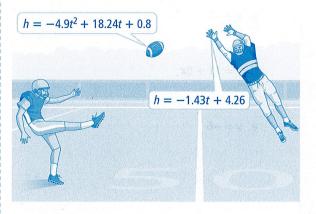


25. Use Structure You are given the system of equations:

$$\begin{cases} y = x + 1 \\ v^2 + x^2 = 2! \end{cases}$$

Solve the system using any of the methods you have learned in this lesson. Explain why you selected the method you used.

26. Reason A football player punts the football, whose path is modeled by the equation $h = -4.9t^2 + 18.24t + 0.8$ for h, in meters, and t, in seconds. The height of a blocker's hands for the same time, t, is modeled as h = -1.43t + 4.26. Is it possible for the blocker to knock down the ball? What else would you have to know to be sure?





ASSESSMENT PRACTICE

27. Classify each function as having exactly one or no points of intersection with the function $y = x^2 + 8x + 11$.

a.
$$y = 2x - 12$$

b.
$$y = 12x + 7$$

c.
$$y = -5$$

d.
$$y = 11 + 8x$$

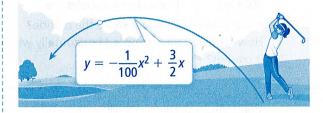
e.
$$y = -6$$

28. SAT/ACT How many solutions does the following system of equations have?

$$\begin{cases} y = 16x - 19 \\ y = 3x^2 + 4x - 7 \end{cases}$$

- A two solutions
- **B** no solutions
- © an infinite number of solutions
- ne solution
- The number of solutions cannot be determined.

29. Performance Task A golfer accidentally hits a ball toward a water hazard that is downhill from her current position on the fairway. The hill can be modeled by a line through the origin with slope $-\frac{1}{8}$. The path of the ball can be modeled by the function $y = -\frac{1}{100}x^2 + \frac{3}{2}x$.



Part A If the golfer stands at the origin, and the water hazard is 180 yd away, will the golfer's ball bounce or splash?

Part B How far did the ball land from the edge of the water hazard?

Part C Does it matter whether you measure the 180 yd horizontally or along the hill? Explain.