

EXPLORE & REASON

Allie spins the spinner and draws one card without looking. She gets a 3 on the spinner and the 3 card. Then she sets the card aside, spins again, and draws another card.



A. Is it possible for Allie to get a 3 on her second spin? On her second card? Explain.

B. Construct Arguments How does getting the 3 card on her first draw affect the probability of getting the 2 card on her second draw? Explain.

HABITS OF MIND

Look for Relationships How are the results from the spinner related to the results from the cards? Explain.



<u> </u>	Assess
Constanting of	9

EXAMPLE 1

EXAMPLE 2

TAALCO

C Try It! Find Probabilities of Mutually Exclusive Events

- 1. A box contains 100 balls. Thirty of the balls are purple and 10 are orange. If you select one of the balls at random, what is the probability of each of the following events?
 - a. The ball is purple or orange.

b. The ball is not purple and not orange.

C Try It! Find the Probabilities of Non-Mutually Exclusive Events

2. A video game is played on a 34 cm by 20 cm rectangular computer screen. A starship is represented by two overlapping circles of radius 6 cm whose area of overlap is 20 cm². A black hole is equally likely to appear at any point on the screen. To the nearest whole percent, what is the probability that the point will appear within the starship?

HABITS OF MIND

Generalize Explain in your own words the meaning of mutually exclusive events. Include examples.



EXAMPLE 3 G Try It! Identify Independent Events

3. There are 10 cards in a box, 5 black and 5 red. Two cards are selected from the box, one at a time.

Assess

a. A card is chosen at random and then replaced. Another card is chosen. Does the color of the first card chosen affect the possibilities of the second card chosen? Explain.

b. A card is chosen at random and *not* replaced. Another card is chosen. Does the color of the first card chosen affect the possibilities of the second card chosen? Explain

Try It! Find Probabilities of Independent Events

4. You spin the spinner two times. Assume that the probability of Blue each spin is $\frac{1}{3}$ and the probability of Orange each spin is $\frac{2}{3}$. What is the probability of getting the same color both times? Explain.



EXAMPLE 4

HABITS OF MIND

Make Sense and Persevere Explain the difference between mutually exclusive events and independent events.

Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How does describing events as independent or mutually exclusive affect how you find probabilities?

Do You KNOW HOW?

5. A bag contains 40 marbles. Eight are green and 2 are blue. You select one marble at random. What is the probability of each event? a. The marble is green or blue.

2. Reason Two marbles are chosen, one at a time, from a bag containing 6 marbles, 4 red marbles and 2 green marbles. Suppose the first marble chosen is green. Is the probability that the second marble will be red greater if the first marble is returned to the bag or if it is not returned to the bag? Explain.

3. Error Analysis The probability that Deshawn plays basketball (event B) after school is 20%. The probability that he talks to friends (event 7) after school is 45%. He says that P(B or T) is 65%. Explain Deshawn's error.

4. Vocabulary What is the difference between mutually exclusive events and independent events?

b. The marble is not green and not blue.

6. A robot at a carnival booth randomly tosses a dart at a square target with 8 inch sides and a circle with a 3 inch radius in the middle. To the nearest whole percent, what is the probability that the dart will land in the circle?

For Exercises 7 and 8, assume that you roll a standard number cube two times.

7. What is the probability of rolling an even number on the first roll and a number less than 3 on the second roll?

8. What is the probability of rolling an odd number on the first roll and a number greater than 3 on the second roll?



UNDERSTAND

- 9. Construct Arguments Let S be a sample space for an experiment in which every outcome is both equally likely and mutually exclusive. What can you conclude about the sum of the probabilities for all of the outcomes? Give an example.
- 11. Higher Order Thinking Murphy's math teacher sometimes wears scarves to class. Murphy has been documenting the relationship between his teacher wearing a scarf and when the class has a math quiz. The probabilities are as follows:
 - P(wearing a scarf) = 10%
 - P(math quiz) = 15%
 - P(wearing a scarf and math quiz) = 5%

Are the events "the teacher is wearing a scarf" and "there will be a quiz" independent events? Explain.

 Error Analysis At Lincoln High School,
 6 students are members of both the Chess Club and the Math Club. There are 20 students in the Math Club, 12 students in the Chess Club, and 400 students in the entire school.

Danielle calculated the probability that a student chosen at random belongs to the Chess Club or the Math Club. Explain her error.

Event C: Student is in Chess Club Event M: Student is in Math Club

$$P(C \text{ or } M) = P(C) + P(M)$$

= $\frac{12}{400} + \frac{20}{400}$
= $\frac{32}{400} = 0.08$

Reason A card is drawn from a box containing 5 cards, each showing a different number from 1 to 5. Consider the events "even number," "odd number," "less than 3," and "greater than 3." Determine whether each pair of events mutually exclusive.

12. < 3, > 3

13. even, > 3

14. odd, > 3

15. odd, even



PRACTICE

16. Hana is playing a virtual reality game in which she must toss a disc to land on the largest triangular section of the board. If the disc is equally likely to land anywhere on the board, what is the probability that she will succeed? Explain. SEE EXAMPLE 1



20. A skydiver is equally likely to land at any point on a rectangular field. Two overlapping circular targets of radius 5 meters are marked on the field. To the nearest percent, what is the probability that the sky diver will land in one or both of the circles? SEE EXAMPLE 2



In a class of 25 students, 8 students have heights less than 65 inches and 10 students have heights of 69 inches or more. For Exercises 17-19, find the probabilities described. SEE EXAMPLE 1

17. P(less than 65 inches or greater than 69 inches)

- 21. Two marbles are chosen at random, one at a time from a box that contains 7 marbles, 5 red and 2 green. SEE EXAMPLES 3 AND 4
 - a. Find the probability of drawing 2 red marbles when the first marble is replaced before the second marble is chosen.

18. P(greater than or equal to 65 inches)

- 19. P(greater than or equal to 65 inches and less than or equal to 69 inches)
- b. Determine whether the situation described is independent.



APPLY

22. Mathematical Connections For a science fair project, Paige wants to test whether ants prefer certain colors. She releases ants on the surface shown, where the circle marked Y is yellow, the circle marked B is blue, and the circle marked W is white. If the ants are randomly distributed across the entire surface, what is the probability that any given ant will be within the blue circle, but not within the yellow circle? Round to the nearest whole percent.



b. The first number is not 7.

c. The first two digits are the same.

d. All three digits are the same.

23. Use Structure A city issues 3-digit license plates for motorized scooters. The digits 0–9 are chosen at random by a computer program. What is the probability that a license plate issued meets each set of criteria?



a. The three-digit number formed is even.

24. Model With Mathematics During a football game, a kicker is called in twice to kick a field goal from the 30 yard line. Suppose that for each attempt, the probability that he will make the field goal is 0.8.

a. What is the probability that he will make both field goals?

b. What is the probability that he will make neither field goal?



ASSESSMENT PRACTICE

- 25. The probability of events A and B both occurring is 15%. The probability of event A or B occurring is 60%. The probability of B occurring is 50%. What is the probability of A occurring?
- 27. Performance Task Paula is packing to visit a friend in another city for a long weekend. She looks at the weather forecast shown below to find the chance of rain. Assume that whether it rains on each day is independent of whether it rains on any other day.



Part A What is the probability that it will not rain on any of the three days to the nearest percent?

26. SAT/ACT A robot spins the spinner shown twice. Assume that the outcomes 1, 2, 3, and 4 are equally likely for each spin. What is the probability that the sum of the two outcomes will be 6?



 $\mathbb{B}\frac{1}{8}$

 $\bigcirc \frac{3}{16}$

 $\mathbb{D}\frac{1}{4}$ $\mathbb{E}\frac{3}{4}$ Part B What is the probability that it will rain at least one of the three days to the nearest percent?

Part C Do you think Paula should pack an umbrella? Explain.





S EXPLORE & REASON

At Central High School, 85% of all senior girls attended and 65% of all senior boys attended the Spring Dance. Of all attendees, 20% won a prize.

A. Assuming that the number of senior girls at Central High School is about equal to the number of senior boys, estimate the probability that a randomly selected senior won a prize at the dance. Explain.

B. Construct Arguments If you knew whether the selected student was a boy or

a girl, would your estimate change? Explain.



HABITS OF MIND

Look for Relationships How would the probability that a senior selected at random won a prize be different if only 60% of senior girls and 50% of senior boys attended the dance? Explain.

EXAMPLE 1 Try It! Understand Conditional Probability 	A second se	Assess
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EXAMPLE 3 G Try It! Apply the Conditional Probability Formula

3. What is the probability that a surveyed student plans to attend but is not a fan of the group?

EXAMPLE 4 🕜 Tr

Notes

Try It! Use Conditional Probability to Make a Decision

4. The marketer also has data from desktop computers. Which product is most likely to be purchased after a related search?

Computer Search and Buying Behavior (% of computer-based site visitors)

Product	Search	Search & Buy
and June 1	35%	10%
K	28%	9%
L	26%	8%
М	24%	5%

HABITS OF MIND

Communicate Precisely Compare the formula used in Example 3, $P(A \text{ and } B) = P(A) \bullet P(B \mid A)$, to the formula used in Example 4, $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$. How are they related? When would you use each formula?





6. Reason At a sports camp, a coach wants to find the probability that a soccer player is a local camper. Because 40% of the students in the camp are local, the coach reasons that the probability is 0.4. Is his conclusion justified? Explain.

554 TOPIC 12 Probability

your reasoning.

W3



UNDERSTAND

- 10. Mathematical Connections How can the formula P(A and B) = P(A) P(B | A) be simplified to find the probability of A and B when the events are independent? Explain.
- 12. Generalize Kiyo is creating a table using mosaic tiles chosen and placed randomly. She is picking tiles without looking. How does P(yellow second | blue first) compare to P(yellow second | yellow first) if the tiles are selected without replacement? If the tiles are selected and returned to the pile because Kiyo wants a different color?



11. Error Analysis From a bag containing 3 red marbles and 7 blue marbles, 2 marbles are selected without replacement. Esteban calculated the probability that two red marbles are selected. Explain Esteban's error.

P(red) = 0.3 $P(red and red) = P(red) \cdot P(red)$ $= 0.3 \cdot 0.3$ = 0.09

- **13.** Use Structure At a fundraiser, a participant is asked to guess what is inside an unlabeled can for a possible prize. If there are two crates of cans to choose from, each having a mixture of vegetables and soup, what is the probability that the first participant will select a vegetable can from the left crate given each situation?
 - a. The left crate has 2 cans of vegetables and 8 cans of soup, and the right crate has 7 cans of vegetables and 3 cans of soup.
 - b. The left crate has 8 cans of vegetables and 2 cans of soup, and the right crate has 5 cans of vegetables and 5 cans of soup.



PRACTICE

For Exercises 14–18, use the data in the table to find the probability of each event. SEE EXAMPLE 1

Technology Class Enrollment by Year

	Sophomore	Junior
Robotics	16	24
Game Design	18	22

14. P(Junior | Robotics)

15. P(Robotics | Junior)

16. P(Game Design | Sophomore)

17. P(Sophomore | Game Design)

- 18. Are year and technology class enrollment dependent or independent events? Explain. **SEE EXAMPLE 2**
- 19. At a high school, 40% of the students play an instrument. Of those students, 20% are freshmen. Of the students who do not play an instrument, 30% are freshmen. What is the probability that a student selected at random is a freshman who plays an instrument? **SEE EXAMPLE 3**

In a study of an experimental medication, patients were randomly assigned to take either the medication or a placebo.

Effectiveness of New Medication As Compared to a Placebo

	Medication	Placebo
Health Improved	53	47
Health Did Not Improve	65	35

20. What is the probability that a patient taking the medication showed improvement? Round to the nearest whole percent. SEE EXAMPLE 1

21. Are taking the medication and having improved health independent or dependent events? SEE EXAMPLE 2

22. Based on the data in the table, would you recommend that the medication be made available to doctors? Explain. SEE EXAMPLE 4



APPLY

- 23. Reason In a recreation center with 1,500 members, 200 are high school students. Of the members, 300 regularly swim. The 45 students of the high school swim team are all members and practice at the pool every week. What is the probability that a high school member selected at random is on the swim team?
- 25. Make Sense and Persevere A box contains 50 batteries, of which 10 are dead and 5 are weak. Suppose you select batteries at random from the box and set them aside for recycling if they are dead or weak. If the first battery you select is dead and the second one is weak, what is the probability that the next battery you select will be weak?

24. Use Structure At the school fair, 5% of students will win a prize. A winner has an equally likely chance to win each prize type shown. What is the probability that a student at the fair will win a comic book? Explain.



26. Higher Order Thinking An inspector at a factory has determined that 1% of the flash drives produced by the plant are defective. If assembly line A produces 20% of all the flash drives, what is the probability that a defective flash drive chosen at random is from the corresponding conveyor belt A? Explain.





S ASSESSMENT PRACTICE

- 27. Which of the following pairs of events are independent? Select all that apply.
 - A student selected at random has a backpack. A student selected at random has brown hair.
 - ^(B) Events A and B, where $P(B \mid A) = \frac{1}{3}$, $P(A) = \frac{3}{5}$ and $P(B) = \frac{5}{9}$
 - © A student selected at random is a junior. A student selected at random is a freshman.
 - D Events A and B, where P(A) = 0.30, *P*(*B*) = 0.25 and *P*(*A* and *B*) = 0.075
 - (E) Events A and B, where P(A) = 0.40, P(B) = 0.3 and P(A and B) = 0.012

28. SAT/ACT The table shows student participation in the newspaper and yearbook by year. A student on the newspaper staff is selected at random to attend a symposium. What is the probability that the selected student is a senior?

Journalism Club Members

	Junior	Senior
Newspaper	16	9
Yearbook	8	17



 $\mathbb{E}\frac{9}{16}$

29. Performance Task In a survey of 50 male and 50 female high school students, 60 students said they exercise daily. Of those students, 32 were female.

Part A Use the data to make a two-way frequency table.

Part B What is the probability that a surveyed student who exercises daily is female? What is the probability that a surveyed student who exercises regularly is male?

Part C Based on the survey, what can you conclude about the relationship between exercise and gender? Explain.





Place Your Guess

A coin toss is a popular way to decide between two options or settle a dispute. The coin toss is popular because it is a simple and unbiased way of deciding. Assuming the coin being tossed is a fair coin, both parties have an equally likely chance of winning.

What other methods could you use to decide between two choices fairly? Think about this during the Mathematical Modeling in 3 Acts lesson.



ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?

2. Write down the main question you will answer about what you saw in the video.

3. Make an initial conjecture that answers this main question.

4. Explain how you arrived at your conjecture.

5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

Develop a Model ACT 2

6. Use the math that you have learned in this Topic to refine your conjecture.

ACT 3 **Interpret the Results**

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?



EXPLORE & REASON

Holly, Tia, Kenji, and Nate are eligible to be officers of the Honor Society. Two of the four students will be chosen at random as president and vice-president. The table summarizes the possible outcomes.

Honor Society Officers

		Vice-	Presi	dent	
		Holly	Tia	Kenji	Nate
resident	Holly	-	HT	HK	HN
	Tia	TH	-	TK	TN
	Kenji	KH	KT	-	KN
•	Nate	NH	NT	NK	-

A. Holly wants to be an officer with her best friend Tia. How many outcomes make up this event?

B. How many outcomes show Holly as president and Tia as vice-president?

C. Generalize How many outcomes have only one of them as an officer? Explain.

HABITS OF MIND

Make Sense and Persevere How could you use the table to calculate the probability that both Holly and Tia will be officers?



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EXAMPI	E 1 C Try It! Use the Fundamental Counting Principle
	 The car that Ms. Garcia is buying comes with a choice of 3 trim lines (standard, sport, or luxury), 2 types of transmission (automatic or manual), and 8 colors. How many different option packages does Ms. Garcia have to choose from? Explain.
,	
EXAMPI	E 2 Solution If y It! Find the Number of Permutations
	 a. Cabriela's 4 favorite congr.
	a. Gabriela's 4 lavorite songs

	b. 5 of the 10 most popular songs

	HABITS OF MIND
	can be used to find the number of ways to arrange 5 different colored beads
	on a sunng.

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EXAMPLE 3 C Try It! Find the Number of Combinations

3. How many ways can a camper choose 5 activities from the 10 available activities at the summer camp?

EXAMPLE 4

Notes

Try It! Use Permutations and Combinations to Find Probabilities

4. Using the data from Example 4, what is the probability that the 5 students' names end with a vowel?

HABITS OF MIND

Look for Relationships What is the relationship between ${}_{10}P_5$ and ${}_{10}C_5$? Is the number of permutations always greater than the number of combinations?



1.9 ESSENTIAL QUESTION How are permutations and combinations useful when finding probabilities?

- 2. Use Structure How is the formula for combinations related to the formula for a permutations?
- 3. Vocabulary Why is it important to distinguish between a permutation and a combination when counting possible outcomes?
- 4. Look for Relationships How is ₉C₂ related to ₉C₇? Explain. How can you generalize this observation for any values of n and r?
- 5. Error Analysis Explain Beth's error.



6. Construct Arguments A company wants to form a committee of 4 people from its 12 employees. How can you use combinations to find the probability that the 4 people newest to the company will be selected?

Do You KNOW HOW?

Do the possible outcomes represent permutations or combinations?

7. Jennifer will invite 3 of her 10 friends to a concert.

Assess

8. Jennifer must decide how she and her 3 friends will sit at the concert.

Find the number of permutations.

9. How many ways can 12 runners in a race finish first, second, and third?

Find the number of combinations.

- 10. In how many ways can 11 contestants for an award be narrowed down to 3 finalists?
- 11. How many different ways can a 4-person team can be chosen from a group of 8 people?

Students will be chosen at random for school spirit awards. There are 6 athletes and 8 non-athletes who are eligible for 2 possible prizes. What is each probability?

12. P(both prizes are awarded to athletes)

- 13. P(both prizes are awarded to non-athletes)
- 14. P(no prize is awarded to an athlete)
- **15.** *P*(no prize is awarded to a non-athlete)
- 16. Explain how Exercises 12 and 13 are similar to Exercises 14 and 15.



UNDERSTAND

- 17. Use Structure Dwayne bought a new bike lock, and the lock came with instructions to choose 3 out of 30 numbers on a circular dial to keep his bike secure. The numbers cannot be repeated. How many possible arrangements can Dwayne choose for his lock? Do the arrangements represent permutations or combinations? Explain.
- 18. Construct Arguments Sage volunteers to read and play with sick children in a hospital. She selects some erasers at random from a bag to use as prizes. There are 8 alien erasers and 10 flying saucer erasers.
 - a. How many groups of 6 erasers can be formed from the 18 erasers? Explain.
 - b. In how many ways can 3 aliens be selected? Explain.
 - In how many ways can 3 aliens and 3 flying saucers be selected? Explain.
 - **d.** What is the probability that 3 aliens and 3 flying saucers will be selected? Explain.

19. Error Analysis There are 6 tiles numbered 1 to 6 in a box. Two tiles are selected at random without replacement to form a 2-digit number. Jeffrey found the probability that the number selected is 16. Explain his error.



20. Mathematical Connections How many lines are determined by the points, *P*, *Q*, *R*, and *S*? Explain.



21. Higher Order Thinking There are 11! different ways for a group of people to sit around a circular table. How many people are in the group? Explain.

PRACTICE

For Exercises 22–27, state if the possible arrangements represent permutations or combinations, then state the number of possible arrangements. SEE EXAMPLES 1, 2, AND 3

- 22. A student chooses at random 4 books from a reading list of 11 books.
- 23. At the end of a season, 10 soccer teams are ranked by the state.
- 24. A committee of 5 people is being selected from a group of 9 to choose the food for a sport's banquet.
- 25. Hugo displays his 8 model planes in a single row.
- 26. A class president, secretary, and treasurer are chosen from 12 students running for office.
- 27. A food truck has a lunch special on tacos. Customers choose a shell, three toppings, and two sides for one price.

Menu Shell: Hard, Soft, Wheat Toppings: Ground Beef, Chicken, Steak, Cheese, Lettuce, **Tomato**. Sour Cream Sides: Fountain Drink, Rice, Beans, Fruit



28. There are 4 comedians and 5 musicians performing in a variety show. The order in which the performers are chosen is random. **SEE EXAMPLE 4**

Practice

a. What is the probability that the first 3 performers are comedians?

b. What is the probability that the first two performers are a comedian followed by a musician?

- 29. A jewelry maker chooses three beads at random from a bag with 10 beads labeled A, B, C, D, E, F, G, H, I, and J. SEE EXAMPLES 2, 3, AND 4
 - a. How can you use permutations or combinations to find P(selected beads spell the initials DEB)? What is the probability?

b. How can you use permutations or combinations to find P(selected beads are all vowels)? What is the probability?





APPLY

- 30. Make Sense and Persevere Amaya's wallet contains three \$1 bills, two \$5 bills, and three \$10 bills. If she pulls 2 bills without looking, what is the probability that she draws a \$1-bill and a \$10-bill? Explain.
- **32.** Look for Relationships Smart Phones, Inc. chooses a 5-digit security code at random from the digits 0–9.
 - a. Suppose the digits cannot be repeated. What is the probability that the security code is 30429? Explain.

- **b.** Suppose the digits can be repeated. What is the probability that the security code is 30429? Explain.
- **31. Model with Mathematics** Raul's favorite restaurant is running a prize game. Five of each of the winning tickets shown are available, and a customer must collect three winning tickets to receive the prize. What is the probability Raul will receive the prize for the baseball cap with his first 3 tickets?



33. Make Sense and Persevere Edwin randomly plays 6 different songs from his playlist.



a. What is the probability that Edwin hears his 6 favorite songs?

b. What is the probability that he hears the songs in order from his most favorite to his sixth most favorite?

S ASSESSMENT PRACTICE

34. Consider an arrangement of 8 items taken 3 at a time in which order is not important. Does each expression give the correct number of arrangements? Select Yes or No.

	Yes	No
₈ P ₃		
₈ C ₃		
8 ^P 3 3!		
8! • 3!		
<u>8!</u> <u>3!</u>		
<u>8!</u> 5!		
<u>8!</u> 3!5!		
8 • 7		

36. Performance Task Use the word shown on the tiles below to find each probability.



Part A Two tiles are chosen at random without replacement. Use conditional probability to find the probability that both letters are vowels. Then find the probability using permutations or combinations. Explain.

- 35. SAT/ACT Fifteen students enter a Safety Week poster contest in which prizes will be awarded for first through fourth place. In how many ways could the prizes be given out?
 - A 4
 - **B** 60
 - © 1,365
 - D 32,760
 - E 50,625

Part B Four of the tiles are chosen at random and placed in the order in which they are drawn. Use conditional probability to find the probability the tiles spell the word SURF. Then find the probability using permutations or combinations. Explain.





EXPLORE & REASON

Mr. and Mrs. Mason have three children. Assume that the probability of having a baby girl is 0.5 and the probability of having a baby boy is also 0.5.





A. Reason Which model represents the situation correctly, Model A or Model B? Explain.

B. What is the probability that Mr. and Mrs. Mason have 3 girls?

C. Compare the probability that the Masons' first child was a boy and they then had two girls to the probability that their first two children were girls and they then had a boy. Does the order affect the probabilities? Explain.

HABITS OF MIND

Look for Relationships Which combinations of children are most common? Is one order of this combination more likely? Explain.

EXAMPLE 1

C Try It! Develop a Theoretical Probability Distribution

1. You select two marbles at random from the bowl. For each situation, define the theoretical probability distribution for selecting a number of red marbles on the sample space {0, 1, 2}. Is it a uniform probability distribution?



Assess

- a. You select one marble and put it back in the bowl. Then you select a second marble.
- b. You select one marble and do not put it back in the bowl. Then you select a second marble.

C Try It! Develop an Experimental Probability Distribution **EXAMPLE 2**

2. Suppose that you selected a student at random from the Drama Club and recorded the student's age.

Ages of Students in Drama Club					
Age	14	15	16	17	18
Students	4	7	10	7	9

a. Define an experimental probability distribution on the sample space {14, 15, 16, 17, 18}.

b. Graph the probability distribution you defined.

		1	

HABITS OF MIND

Look for Relationships Compare the graph of a uniform probability distribution with the graph of a non-uniform distribution.

 EXAMPLE 3 S Try It! Binomial Experiments Is the experiment a binomial experiment? If so, find the probability of success. Explain. You select one card at random from a set of 7 cards, 4 labeled A and 3 labeled B. Then you select another card at random from the cards that remain. For each selection, success is that the card is labeled A. You roll a standard number cube 4 times. Assume that each time you roll the number cube, each number is equally likely to come up. For each roll, success is getting an even number. 	Notes		Assess
 3. Is the experiment a binomial experiment? If so, find the probability of success. Explain. a. You select one card at random from a set of 7 cards, 4 labeled A and 3 labeled B. Then you select another card at random from the cards that remain. For each selection, success is that the card is labeled A. b. You roll a standard number cube 4 times. Assume that each time you roll the number cube, each number is equally likely to come up. For each roll, success is getting an even number. EXAMPLE 4 S Try It! Probabilities in a Binomial Experiment 4. To the nearest tenth of a percent, what is the probability that Terrell has more than 3 winning cards? Explain. 		EXAMPLE 3	Try It! Binomial Experiments
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HABITS OF MIND Use Structure Explain why $_nC_r$ appears in the formula for the probability of a binomial experiment.			4. To the nearest tenth of a percent, what is the probability that Terrell has more than 3 winning cards? Explain.
HABITS OF MIND Use Structure Explain why ${}_{n}C_{r}$ appears in the formula for the probability of a binomial experiment.			
HABITS OF MIND Use Structure Explain why $_nC_r$ appears in the formula for the probability of a binomial experiment.			
HABITS OF MIND Use Structure Explain why ${}_{n}C_{r}$ appears in the formula for the probability of a binomial experiment.			
Use Structure Explain why ${}_{n}C_{r}$ appears in the formula for the probability of a binomial experiment.			HABITS OF MIND
			Use Structure Explain why $_nC_r$ appears in the formula for the probability of a binomial experiment.

Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION What are the characteristics of a binomial experiment?

Do You KNOW HOW?

Graph the probability distribution P.

- 4. Theoretical probabilities from selecting a student at random from a group of 3 students, Jack, Alani, and Seth
- 5. Probabilities from flipping a fair coin 3 times and counting the number of heads. The sample space is the set of numbers 0, 1, 2, 3. P(0) = 0.125, P(1) = 0.375, P(2) = 0.375,P(3) = 0.125

A bag contains 5 balls: 3 green, 1 red, and 1 yellow. You select a ball at random 4 times, replacing the ball after each selection. Calculate the theoretical probability of each event to the nearest whole percent.

6. getting a green ball exactly 3 times

7. getting a green ball exactly 4 times

8. getting a green ball at least 3 times

9. getting a yellow ball twice

10. getting only red and green balls

2. Vocabulary What is the difference between a binomial experiment and one that is not binomial?

3. Error Analysis A regular tetrahedron has four triangular sides, with one of the letters A, B, C, and D on each side. Assume that if you roll the tetrahedron, each of the letters is equally likely to end up on the bottom. {A, B, C, D} is a sample space for the experiment. Rochelle was asked to find the theoretical probability distribution for the experiment. Explain and correct the error.







UNDERSTAND

- **11. Communicate Precisely** Explain what it means for a coin to be a fair coin.
- 14. Error Analysis A standard number cube is rolled 7 times. Success for each roll is defined as getting a number less than 3. Abby tried to calculate the probability of 5 successes. Describe and correct her error.

$$P(5) = \left(\frac{1}{3}\right)^{5} \left(\frac{2}{3}\right)^{2} \approx 0.002$$

12. Reason You spin the spinner shown.



Describe a theoretical probability distribution for the experiment.

13. Communicate Precisely Five students in a

class of 27 students ate hamburgers for lunch. Suppose the teacher selects a student in the class at random and then selects another student at random. Success for each selection **15. Mathematical Connections** A marble is selected from the bowl shown 4 times. The marble is returned to the bowl after each selection.



a. Show that there are exactly ₄C₂ ways to get exactly 2 green marbles.

b. How are ${}_{5}C_{3}$ and ${}_{5}C_{2}$ related? Explain.

is selecting a student who ate a hamburger. Is this a binomial experiment? Explain.



PRACTICE

A card is chosen at random from the box containing 10 cards: 3 yellow, 4 red, 2 green, and 1 blue. SEE EXAMPLES 1 AND 2

- 16. Define a probability distribution for this experiment on the sample space {Y, R, G, B}.
- 21. There are 10 black and 10 red cards face down on the table. One card is selected at random. Then another card is selected at random. Success is getting a red card.

17. Graph the probability distribution.

In a certain game, the player can score 0, 1, 2, 3, or 4 points during their turn. The table shows the number of times Kennedy scored each number of points the last time she played the game. **SEE EXAMPLE 2**

Score	0	1	2	3	4
Frequency	3	7	9	6	5

- 18. Define an experimental probability distribution based on Kennedy's scores.
- 19. Graph the probability distribution you defined in Exercise 18.

Is the experiment a binomial experiment? Explain. **SEE EXAMPLE 3**

20. A quality control specialist tests 50 LED light bulbs produced in a factory. Success is that a tested light bulb burns for at least 2,000 hours without dimming. For each light bulb, the probability of success is 0.9.

22. A basketball player is shooting 2 free throws. The probability of her making the first free throw is 0.86. The probability of her making

the second free throw is 0.92.

Each time Bailey is at bat, the probability that he gets a hit is 0.250. If he bats 10 times in the course of two games, what is the probability of each result? Round to the nearest tenth of a percent. SEE EXAMPLE 4

23. He gets no hits.

24. He gets exactly 1 hit.

25. He gets exactly 2 hits.

26. He gets fewer than 3 hits.





APPLY

27. Model with Mathematics The circle graph shows the result of a survey of the most popular types of music in the U.S., based on sales, downloads, and streaming.



a. Define a probability distribution for the sample space.

- **28. Higher Order Thinking** A pharmaceutical company is testing a new version of a medication. In a clinical trial of the old version of the medication, 18% of the subjects taking the old medication experienced headaches.
 - a. Suppose that 18% of the people taking the new medications will experience headaches.
 If 8 subjects are selected at random and given the new medication, what is the probability that less than two of them will experience headaches?

b. Suppose that two of the eight subjects experience headaches after taking the new medication. Is that cause for concern? Explain your reasoning.

b. Graph the probability distribution.

c. According to the survey, which is the most popular type of music in the United States?

29. Communicate Precisely In a quiz show, a contestant is asked 6 questions. Each question has 5 answer choices. Assume that the contestant picks an answer at random for each question and the probability of guessing the correct answer is 20%. What is the probability of guessing correctly on at least 4 of the questions? Round your answer to the nearest tenth of a percent.



S ASSESSMENT PRACTICE

30. You are going to roll a

game piece two times. The game piece has 10 sides of equal area, each with one of the numbers 0 through 9. Assume that it is equally likely to land with any of



the sides on top. Success is defined as getting a 3 on top.

Let P be the function defined on {0, 1, 2} such that P(n) is the probability of *n* successes. Select all that apply.

- (A) This is a binomial experiment.
- ^(B) *P* is a probability distribution for the sample space {0, 1, 2}.
- $\bigcirc P(0) = 0.81$
- (D) P(1) = 0.09
- (E) P(2) = 0.01

- 31. SAT/ACT A standard number cube is rolled 6 times. Success is defined as getting a number greater than 4. Rounded to the nearest percent, what is the probability of exactly 2 successes?
 - A 2%
 - **B** 8%
 - C 23%
 - D 33%
 - E 50%

32. Performance Task Get 5 index cards. Draw a picture on one side and no picture on the other side of each card.

Part A You are going to throw all 5 cards up in the air and count the number of cards that land face up. Assume that it is equally likely that each card will land face up and face down. Define a theoretical probability distribution for the sample space {0, 1, 2, 3, 4, 5}.

Part B Perform the experiment 20 times. Each time you perform the experiment, record the number of cards that land face up. Find the experimental probability for each outcome in the sample space {0, 1, 2, 3, 4, 5} and define an experimental probability distribution the sample space.

Part C Compare the results of Part A and B. If they are different, explain why you think they are different.





EXPLORE & REASON

A company has 20 employees whose hourly wages are shown in the bar graph.



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12 - 5

Expected Value

A. An employee is chosen at random. What is the probability that his or her hourly wage is \$12? \$25? \$50?

B. What is the mean hourly wage? Explain your method.

c. Construct Arguments Is the mean a good description of the typical hourly wage at this company? Explain.

HABITS OF MIND

Reasoning Compare the mean hourly wage to the median hourly wage. Which would be a more useful value to know if you want to estimate the total amount the company pays its employees? Explain.



Notes		Assess
	EXAMPLE 1	Try It! Evaluate and Apply Expected Value
· · · · · · · ·		 a. What would happen to the expected value if fewer people ordered chili and more people ordered stew? Explain.
2 E E E E 2 2 2 E E E E 2 2 2 X X X Z Z		b. Suppose the restaurant's profit on an order of stew increased by \$0.05
		and the profit on an order of chili decreased by \$0.05. How would these changes affect the expected profit per meal?
		· · · · · · · · · · · · · · · · · · ·
	EXAMPLE 2	Try It! Find Expected Payoffs
		2. What is the expected payoff for the person making the donation?
	* * * * * * * * *	
* * * * * *		
	5 c c c 2 5 c c	HABITS OF MIND
		Construct Arguments What happens to expected value when the value of an
		outcome increases while its probability decreases?



EXAMPLE 3 Contraction Try It! Use Expected Values to Evaluate Strategies

3. The insurance company in Example 3 also offers optional safety glass coverage. Annual windshield repair statistics are: 50% no repairs, 30% minor repairs (\$50), and 20% full replacement (\$300). Should OptionPremium
(\$)DeductibleDeductible50200No Deductible1000

Assess

the owner opt for safety glass coverage? Which option has the lower expected cost?

EXAMPLE 4

C Try It! Use Binomial Probability to Find Expected Value

4. A carnival game has 4 orange lights and 1 green light that flash rapidly one at a time in a random order. When a player pushes a button, the game stops, leaving one light on. If the light is green, the player wins a prize. Copy and complete the table, then determine the number of prizes that a player can expect to win if the game is played 4 times.

Number of Green Lights (wins)	Probability
	$_4C_0(0.2)^0(0.8)^4 = $
	$_4C_{(0.2)}(0.8)^{(0.8)} = 10$
2	$C_{(0.2)}(0.8) = 1$
3	$C_{\rm m}(0.2)^{\rm m}(0.8)^{\rm m} =$
4	$C_{-}(0.2)^{-}(0.8)^{-}=$

HABITS OF MIND

Generalize When do you add expected values and when do you compare individual expected values? Explain.



Do You UNDERSTAND?

1.2 ESSENTIAL QUESTION What does expected value tell about situations involving probability?

2. Error Analysis Benjamin is finding the expected value of the number of heads when tossing a fair coin 10 times. What is Benjamin's error?

Toss a coin 10 times E = 50%

3. Construct Arguments A carnival game costs \$1 to play. The expected payout for each play of this game is \$1.12. Should the carnival operators modify the game in way? Explain.

- 4. Reason The students in Ms. Kahn's class are raising money to help earthquake victims. They expect to raise \$0.52 for each raffle ticket they sell. If each raffle ticket is sold for \$2, what can you conclude?
- **5. Vocabulary** A spinner is divided into 6 equalsized sectors, numbered 1, 1, 1, 4, 7, and 10. Is the expected value of a spin the same as the mean of the numbers? Explain.

Do You KNOW HOW?

- 6. What is the expected value when rolling a standard number cube?
- 7. What is the expected value when rolling two standard number cubes?
- 8. A travel website reports that in a particular European city, the probability of rain on any day in April is 40%. What is the expected number of rainy days in this city during the month of April?
- 9. You buy an airplane ticket for \$900. You discover that if you cancel or rebook your vacation flight to Europe, you will be charged an extra \$300. There is a 20% chance that you will have to rebook your flight.
 - a. What is the expected value of the cost of the ticket?
 - **b.** Is the expected value the amount you will pay to book the ticket whether or not you have to rebook? Explain.
- 10. A child-care service charges families an hourly rate based upon the age of the child. Their hourly rate per child is \$20 per hour for infants less than 1 year old, \$18 for toddlers 1 to 3 years old, \$15 per hour children 3 or more years old. The ratio of infants : toddlers : 3+ years is 2 : 3 : 5. What is the expected charge per child per hour?



UNDERSTAND

11. Error Analysis For the dartboard shown, Deshawn calculated the expected number of points per dart. Explain Deshawn's error. What is the correct expected value?



Expected value $=\frac{2}{7}(4) + \frac{5}{7}(1)$ $= \frac{8}{7} + \frac{5}{7} \\ = \frac{13}{7} \approx 1.86$

12. Reason A nonrefundable plane ticket costs
\$600, while a refundable ticket costs \$900.
A traveler estimates there is a 20% chance he will have to cancel his upcoming trip.
Should the traveler purchase a refundable or nonrefundable ticket? Explain.

13. Construct Arguments A consumer determines that her expected cost for Option B is \$528 per year.

Option	Annual Premium	Deductible
А	\$600	\$0
В	\$500	\$1,000

a. Why might this consumer select the policy with the \$1000 deductible?

b. Why might this consumer select the policy with no deductible?

14. Mathematical Connections How is expected value related to the mean?



PRACTICE

A farmer estimates her hens will produce 3,000 dozen more eggs this year than last year. She estimates the probability of her net profit or loss on each dozen eggs based on her costs. **SEE EXAMPLE 1**



15. What is her expected profit per dozen eggs?

18. An insurance company offers three policy options. The probability a car will be damaged in a given year is 5%, and if a car is damaged, the cost of the repairs will be \$1000. Which option has the least expected annual cost for the car owner? Explain. SEE EXAMPLE 3

Insurance	Policy	Options
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Option	Annual Premium (\$)	Deductible (\$)
А	900	0
В	800	400
С	700	1000

On a tropical island, the probability of sunny weather is 90% each day. SEE EXAMPLE 4

- 19. What is the expected number of sunny days in a non-leap year?
- production?

16. What is her expected profit on the total egg

17. An electronics store offers students a discount of 10% on purchases of computers. They estimate that $\frac{1}{16}$ of computer sales are to students. The average sale per customer is \$498 and the store's profit is \$80 before the discount. What is the expected profit on the sale of a computer? SEE EXAMPLE 2

20. What is the expected number of sunny days during the month of June?



APPLY

- 21. Model With Mathematics A solar panel company has found that about 1% of its panels are defective. The company's cost to replace each defective panel is \$600. A consultant recommends changes to the manufacturing process that will cost \$200,000 and reduce the defective rate to 0.2%. The company estimates that it will sell 30,000 panels next year and that sales will increase by 5,000 panels per year for the next 10 years. Should the company follow the consultant's recommendation? Explain.
- 23. Higher Order Thinking Your family is going to buy a new TV set for \$599. You find out that the probability that the TV set will need to be serviced in the second year is 0.05 and the probability that the TV set will need to be serviced in the third year is 0.08. A 2-year warranty costs \$55, and a 3-year warranty costs \$80. The average cost of repairing the TV set is \$278. What would you advise your family to do, get a 2-year extended warranty, a 3-year extended warranty or not to get any extended warranty? Explain your reasoning.

- 22. Reason A student tosses a coin 4 times and the results are heads, tails, heads, and heads. The student concludes that the expected number of heads for 100 tosses is 75. How did the student find this number? Do you agree with the student's reasoning? Explain.
- 24. Make Sense and Persevere A company makes tablets that are guaranteed for one year. On average, one out of every 200 tablets needs to be repaired or replaced within the first year. If a tablet needs to be repaired, the company loses an average of \$140. If the company sells 2,600,000 of the tablets in a year, what is their net profit on the sale of the tablets in that year?



If no repairs or replacement is needed for a tablet, the company makes a \$24 profit on that tablet.

ASSESSMENT PRACTICE

25. A commuter recorded data on the arrival time of his morning train each weekday for 5 weeks. According to the data, he should expect the train to be 1.16 minutes late on any given day. What are the missing values in the commuter's table?

Arrival Time for Train

Minutes late	0	1	2	3	4	5
Number of days		5	1		1	3

Part A Write design instructions for the board so that a chip tossed randomly onto the board is 8 times more likely to land in the outer region than in the inner square. Explain your reasoning.

Part B Assign a whole number of points to the outer region so that the expected score on a single toss is as close as possible to 5. Explain your reasoning.

26. SAT/ACT What is the expected total for 20 spins?

A 100

B 105

© 110

D 115

E 120



27. Performance Task A toy company is designing a children's game in which players toss chips onto a board. The square board will contain a smaller square at its center.



Part C If the area of the inner square is doubled and the overall size of the board remains the same, how does the expected score change? Is it also doubled? Explain.





CRITIQUE & EXPLAIN

Your friend offers to play the following game with you. "If the product of the rolls of two number cubes is 10 or less, I win. If not, you win!"



A. If you were to play the game many times, what percent of games would you expect to win?

B. Is the game fair? Should you take the offer? Explain.

C. Make Sense and Persevere Suggest a way to change the game from fair to unfair, or vice versa, while still using the product of the two number cubes. Explain.

HABITS OF MIND

Use Structure Change the game from using products to using a different mathematical number operation. Can you make the game fair? Explain your reasoning.

12 - 6

Probability and



Assess

EXAMPLE 1

EXAMPLE 2

NOTES

C Try It! Use Probability to Make Fair Decisions

1. Your trainer creates training programs for you. How can you use index cards to randomly choose the following: Strength training 1 day per week; Cardio training 2 days per week, with no consecutive days; Swimming 1 day per week.

C Try It! Determine Whether a Decision Is Fair or Unfair

- 2. Justice and Tamika use the same 3 cards, but change the game. In each round, a player draws a card and replaces it, and then the other player draws. The differences between the two cards are used to score each round. Order matters, so the difference can be negative. Is each game fair? Explain.
 - a. If the difference between the first and second cards is 2, Justice gets a point. Otherwise Tamika gets a point.

b. They take turns drawing first. Each round, the first player to draw subtracts the second player's number from her own and the result is added to her total score.

	Assess
EXAMPLE 3	Y ITY It! Make a Decision Based on Expected Value
	 Additional data is collected for the TAB5000 and TAB5001. The manufacturing cost and the replacement cost for the TAB5001 remain
	unchanged.
	a. The production and replacement costs for the TAB5000 increased by \$10. What would the expected profit be for the TAB5000?
[
	b. The failure rate for the TAB5001 increased by 1%. What would the
	expected profit be for the TABS001?
	c. As a consultant for the company, what would you recommend they do to maximize their profit?
	HADITS OF MIND
	IADI 15 UF MIND
	Construct Arguments When do you need to compute and compare expected values instead of just comparing probabilities? Explain.
ere contra con por ele contra de ele	
EYAMDI E A	Try Itl. Use a Rinomial Distribution to Make Decisions
EXAMIFLE 4	II y It. Ose a Difformal Distribution to Make Decisions
	4. A play calls for a crowd of 12 extras with non-speaking parts. Because 10%
	of the extras have not shown up in the past, the director selects 15 students
	as extras. Find the probabilities that 12 extras show up to the performance,
	15 extras show up to the performance, and more than 12 extras show up to
	the performance.
la e e e e e e e é é é é e e e e	
a e e e se e e s'a are proporte e e	
	HABITS OF MIND
	Use Structure What three expressions are multiplied together in the binomial probability formulaand what do they represent?



Do You UNDERSTAND?

1.? ESSENTIAL QUESTION How can you use probability to make decisions?

2. Reason How can you use random numbers to simulate rolling a standard number cube?

Do You KNOW HOW?

7. A teacher assigns each of 30 students a a unique number from 1 to 30. The teacher uses the random numbers shown to select students for presentations.



Assess

Which student was selected first? second?

3. Error Analysis Explain the error in Diego's reasoning.

If a game uses random numbers, it is always fair. 8. Three friends are at a restaurant and they all want the last slice of pizza. Identify three methods involving probability that they can use to determine who gets the last slice. Explain mathematically why each method will guarantee a fair decision.

- 4. Use Structure Describe what conditions are needed for a fair game.
- **9.** Edgar rolls one number cube and Micah rolls two. If Edgar rolls a 6, he wins a prize. If Micah rolls a sum of 7, she gets a prize. Is this game fair? Explain.

- Use Appropriate Tools Explain how you can visualize probability distributions to help you make decisions.
- 6. Reason Why must the expected value of a fair game of chance equal zero?
- 10. The 10 parking spaces in the first row of the parking lot are reserved for the 12 members of the Student Council. Usually an average of ten percent of the Student Council does not drive to school dances. What is the probability that more members of the Student Council will drive to a dance than there are reserved parking spaces?





UNDERSTAND

 Reason Suppose Chris has pair of 4-sided dice, each numbered from 1 to 4, and Carolina has a pair of 10-sided dice, each numbered from 1 to 10. They decide to play a series of games against each other, using their own dice.



a. Describe a game that would be fair. Explain.

b. Describe an unfair game. Explain.

12. Construct Arguments Mr. and Ms. Mitchell have 3 children, Luke, Charlie, and Aubrey. All 3 children want to sit in the front seat. Charlie suggests that they flip a coin two times to decide who will sit in the front seat. The number of heads determines who sits in the front seat. Is this a fair method? Explain.

Number of Heads	Front Seat Passenger
0	Luke
1	Charlie
2	Aubrey

13. Error Analysis Mercedes is planning a party for 10 people. She knows from experience that about 20% of those invited will not show up. If she invites 12 people, how can she calculate the probability that more than 10 people will show up. What error did she make? What is the correct probability?

> Use the binomial distribution for 12 trials, with a 20% probability, and more than 10 show up. $(12)(0.80)^{1}(0.20)^{11} +$ $(1)(0.80)^{0}(0.20)^{12}$



PRACTICE

14. How can you use random integers to select 3 students from a group of 8 to serve as student body representatives, so that each student is equally likely to be selected? **SEE EXAMPLE 1**

Fatima is a contestant on a game show. So far, she has won \$34,000. She can keep the \$34,000 or spin the spinner shown below and add or subtract the amount shown from \$34,000. SEE EXAMPLE 3



- 17. If Fatima spins the spinner, what are her expected total winnings?
- 18. Would you advise Fatima to keep the \$34,000 or to spin the spinner? Explain your reasoning.
- 19. Suppose 0.5% of people who file federal tax returns with an adjusted gross income (AGI) between \$50,000 and \$75,000 are audited. Of 5 people in that tax bracket for whom ABC Tax Guys prepared their taxes, 2 were audited. SEE EXAMPLE 4
 - a. If 5 people with an AGI between \$50,000 and \$75,000 are selected at random from all the people who filed federal tax returns, what is the probability that at least 2 people are audited?
 - b. Would you recommend that a friend with an AGI between \$50,000 and \$75,000 use ABC Tax Guys to prepare her tax returns? Explain.

- Explain whether each game is fair or unfair. **SEE EXAMPLE 2**
- 15. When it is your turn, roll a standard number cube. If the number is even, you get a point. If it is odd, you lose a point.

16. When it is your turn, roll two standard number cubes. If the product of the numbers is even, you get a point. If the product is odd, you lose a point.

590 TOPIC 12 Probability



APPLY

20. Model With Mathematics For \$5.49 per month, Ms. Corchado can buy insurance to cover the cost of repairing a leak in the natural gas lines within her house. She estimates that there is a 3% chance that she will need to have such repairs made next year.



a. What is the expected cost of a gas leak, if Ms. Corchado does not buy insurance? Use the cost shown in the middle of the graph.

b. With more recent information, Ms. Corchado learns that repair costs could be as much as \$1,200 dollars with an 8% probability of a leak. What is the expected cost of a gas leak with these assumptions?

c. Would you advise Ms. Corchado to buy the insurance? Explain.

- 21. Higher Order Thinking You are a consultant to a company that manufactures components for cell phones. One of the components the company manufactures has a 4% failure rate. Design changes have improved the quality of the component. A test of 50 of the new components found that only one of the new components is defective.
 - a. Before the design improvements what was the probability that among 50 of the items, at most one of the items was defective?

b. Is it reasonable to conclude that the new components have a lower failure rate than 4%?

c. Would you recommend further testing to determine whether the new parts have a lower failure rate than 4%? Explain.

$\mathbf{\mathbf{S}}$ **ASSESSMENT PRACTICE**

- 22. Paula, Sasha, and Yumiko live together. They want a system to determine who will wash the dinner dishes on any given night. Select all of the methods that are fair.
 - A Roll a standard number cube. If the result is 1 or 2, Paula does the dishes; if 3 or 4, Sasha; if 5 or 6, Yumiko.
 - ^(B) Roll a standard number cube. If the result is 1, Paula does the dishes; if 2, Sasha; if 4, Yumiko. If the result is 3, 5, or 6, roll again.
 - © Roll two standard number cubes. If the sum of the numbers that come up is less than 6, Paula washes the dishes; if the sum is 8, 9, or 12, Sasha; if the sum is 6 or 7, Yumiko. If the sum is 10 or 11, roll again.
 - D Write the name of each girl on a slip of paper, place the slips in a box, mix them up, and select one at random. The person whose name is selected does the dishes.

24. Performance Task Acme Tire Company makes two models of steel belted radial tires, Model 1001 and Model 1002.



If one of these tires fails and the company is sued, the average settlement is \$1,200,000.

Part A Find the expected profit for both models of tires after any potential lawsuits. Explain.

- 23. SAT/ACT A fair choice among a group of students may be made by flipping three coins in sequence, and noting the sequences of heads and tails. If each student is assigned one of these sequences, how many students can be selected fairly by this method?
 - **A**4
 - **B** 5
 - © 6
 - **D**7
 - **E** 8

Part B Would you recommend that the company continue selling both models? Explain.