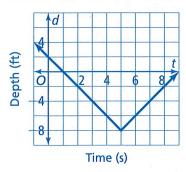


EXPLORE & REASON

A diver is doing ocean search-and-rescue training. The graph shows the relationship between her depth and the time in seconds since starting her dive.







- C. What is the average speed of the diver's descent? How can you tell from the graph?
- D. Communicate Precisely What does the V-shape of the graph tell you about the dive? What information does it not tell you about the dive?

HABITS OF MIND

Reason What do the points where the graph intersects the x-axis tell you about the dive?



EXAMPLE 1

- Try It! Understand Domain and Range
 - 1. What are the domain and range of each function? Write the domain and range in set-builder notation and interval notation.
 - a. y = |x 4|

b.
$$y = 6x - 2x^2$$

EXAMPLE 2



Try It! Find x- and y-intercepts

2. What are the x- and y-intercepts of $g(x) = 4 - x^2$?

HABITS OF MIND

Make Sense and Persevere A function does not have any x-intercepts. What might be true about its domain and range?

EXAMPLE 3



Try It! Identify Positive or Negative Intervals

3. a. For what interval(s) is the function h(x) = 2x + 10 positive?

b. For what interval(s) is the function negative?





EXAMPLE 4

Try It! Identify Where a Function Increases or Decreases

4. For what values of x is each function increasing? For what values of x is each function decreasing?

a.
$$f(x) = x^2 - 4x$$

b.
$$f(x) = -2x - 3$$

HABITS OF MIND

Use Structure Can a function be increasing and negative on the same interval? Explain.

EXAMPLE 5



Try It! Understand Average Rate of Change Over an Interval

5. What do the average rates of change of the function y = |x| + 2 over the intervals [-2, 0], [0, 3], and [-2, 3] indicate about the function?

HABITS OF MIND

Construct Arguments If a function has a positive average rate of change over an interval, does that mean that the function must be increasing over that interval? Explain.

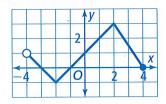


Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How do graphs and equations reveal information about a relationship between two quantities?

Do You KNOW HOW?

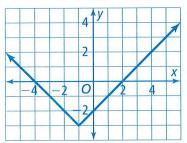
Find each key feature.



4. domain

- 2. Vocabulary Define the term zero of a function in your own words.
- 5. range
- 6. x-intercept(s)
- 7. y-intercept(s)
- 8. interval(s) where the graph is positive
- 9. interval(s) where the graph is decreasing
- 10. interval(s) where the graph is increasing
- 11. rate of change on [-1, 4]

3. Error Analysis Lonzell said the function shown in the graph is positive on the interval (-1, 5)and negative on the interval (-5, -1).



Identify and correct Lonzell's error.

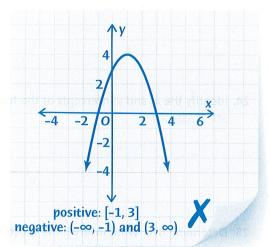




UNDERSTAND

12. Reason The graph of $y = -\frac{1}{2}x + 2$ is negative over the interval (4, ∞) and positive over the interval $(-\infty, 4)$. What happens on the graph when x = 4? Explain.

13. Error Analysis Describe and correct the error a student made in finding the interval(s) over which the function is positive and negative.



14. Use Structure Sketch a graph given the following key features.

domain: (-4, 4)

range: (-4, 6]

increasing: (-4, 1)

decreasing: (1, 4)

x-intercepts: (-2, 0), (3, 0)

y-intercept: (0, 4)

negative: (-4, -2) and (3, 4) positive: (-2, 3)

15. Construct Arguments A student says that all linear functions are either increasing or decreasing. Do you agree? Explain.

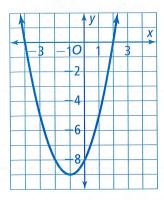
16. Higher Order Thinking A relative maximum of a function occurs at the highest point on a graph over a certain interval. A relative minimum of a function occurs at the lowest point on a graph over a certain interval. Explain how to identify a relative maximum and a relative minimum of a function using key features.

17. Model With Mathematics For a graph of speed in miles per hour as a function of time in hours, what does it mean when the function is increasing? Decreasing?



PRACTICE

Use the graph of the function for Exercises 18–22.



18. Identify the domain and range of the function. SEE EXAMPLE 1

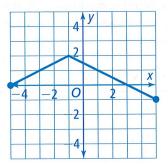
19. Identify the x- and y-intercepts of the function. SEE EXAMPLE 2

20. On what intervals is the function positive? On what intervals is it negative? SEE EXAMPLE 3

21. On what intervals is the function increasing? On what intervals is it decreasing? SEE EXAMPLE 4

22. What is the average rate of change over the interval (-3, 2)? SEE EXAMPLE 5

Use the graph of the function for Exercises 23-27.



23. Identify the domain and range of the function. SEE EXAMPLE 1

24. Identify the x- and y-intercepts of the function. **SEE EXAMPLE 2**

25. Determine over what interval the function is positive or negative. SEE EXAMPLE 3

26. Determine over what interval the function is increasing or decreasing. SEE EXAMPLE 4

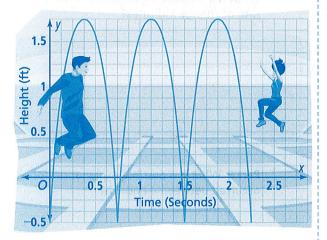
27. What is the average rate of change over the interval (-1, 5)? SEE EXAMPLE 5



APPLY

- 28. Communicate Precisely Kathryn is filling an empty 100 ft³ container with sand at a rate of 1.25 ft³/min. Describe the key features of the graph of the amount of sand inside the container.
- c. Over what intervals is the graph negative? Explain what the negative intervals represent.

29. Make Sense and Persevere The graph shows a jumper's height, y, in feet x seconds after getting onto a trampoline.



of the average rate of change.

30. Model With Mathematics Bailey starts playing a game on her cell phone with the battery fully charged, and plays until the phone battery dies. While playing the game, the charge in Bailey's battery decreases by half a percent

d. What is the average rate of change over the interval [0.75, 1.125]? Explain the meaning

- a. What are the x- and y-intercepts? Explain what the x- and y-intercepts represent.
- a. Write a function for the percent charge in the battery while Bailey is playing the game.

per minute.

b. Over what intervals is the graph positive? Explain what the positive intervals represent.

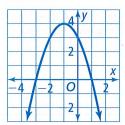
b. What is the domain and range of the function?

c. How long can Bailey play the game?



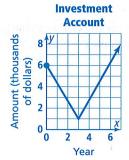
(S) ASSESSMENT PRACTICE

31. Given the graph, select yes or no for each statement.



	Yes	No
a. The domain is $(-\infty, 4]$.	0	0
b. The range is (-∞, 4].	0	0
c. The graph is positive on the interval (0, ∞).	0	0
d. The graph is decreasing on the interval (–1, ∞).	O .	0

32. SAT/ACT The graph shows the amount of money in an investment account. Which statement is true?



- (A) \$6,000 was initially invested in the account.
- ® \$1,000 was initially invested in the account.
- © At Year 3, there was \$0 in the account.
- ① At Year 7, there was \$0 in the account.

33. Performance Task The graph shows the amount of water in a water tank over several hours.



Part A What is the average rate of change on the interval [0, 4] and on the interval [6, 10]? What is a possible explanation for what each rate of change indicates?

Part B What is a possible explanation for what occurred between 4 and 6 h?

Part C What is the average rate of change on the interval [0, 10]? What does the rate of change mean? Does this rate of change give a good indication as to what is happening with the water in the cistern from 0 h to 10 h? Explain.



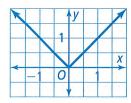
Transformations

of Functions

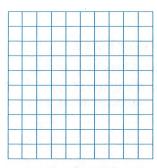
PearsonRealize.com

EXPLORE & REASON

The graph of the function f(x) = |x| is shown.



A. Graph the function g(x) = |x + c| for each of several different values of c between -5 and 5.



B. Look for Relationships Predict what will happen to the graph if c is a number greater than 100. What if c is a number between 0 and $\frac{1}{2}$?

HABITS OF MIND

Reason What happens to the graph if c is a negative number?

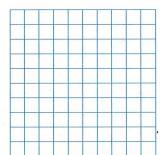
EXAMPLE 1 Try It! Translate a Function

1. a. How did the transformation of f to g in part (a) affect the intercepts?

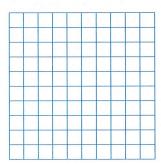
b. How did the transformation of f to g in part (b) affect the intercepts?

EXAMPLE 2 Try It! Reflect a Function Across the x- or y-Axis

- 2. What is an equation for the reflected graph? Check by graphing.
 - a. the graph of $f(x) = x^2 2$ reflected across the x-axis.



b. the graph of $f(x) = x^2 - 2$ reflected across the *y*-axis.

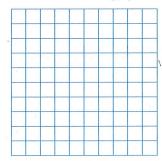


HABITS OF MIND

Look for Relationships How are the intercepts of a graph affected by reflection across the *x*-axis? Explain.

EXAMPLE 3 Try It! Understand Stretches and Compressions

3. Show that $j(x) = f(\frac{1}{2}x)$ is a horizontal stretch of the graph of f.

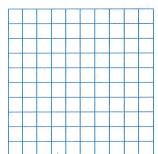




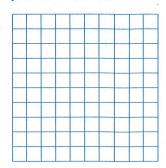
EXAMPLE 4

Try It! Graph a Combination of Transformations

- **4.** Using the graph of f in Example 4, graph each equation.
 - a. y = f(2x) 4



b.
$$y = f(2x - 3) - 2$$



HABITS OF MIND

Model With Mathematics If the graph of a parent function is vertically stretched by a factor of 2 and then translated 3 units down, would you get the same graph if you translated the parent graph 3 units down first and then vertically stretched it by a factor of 2? Explain.

EXAMPLE 5



Try It! Identify Transformations From an Equation

5. What transformations of the graph of f(x) = |x| are applied to graph the function g?

a.
$$g(x) = \frac{1}{2}|x+3|$$

b.
$$g(x) = -|x| + 2$$

EXAMPLE 6



Try It! Write an Equation From a Graph

6. How would the graph and equation be affected if the train traveled twice as far in the same amount of time?

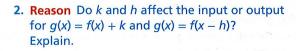
HABITS OF MIND

Make Sense and Persevere The function f(x) = |x| is translated 3 units right and 2 units down, and then vertically stretched by a factor of 4. What is an equation for the transformed function g?



Do You UNDERSTAND?

1.? ESSENTIAL QUESTION What do the differences between the equation of a function and the equation of its parent function tell you about the differences in the graphs of the two functions?



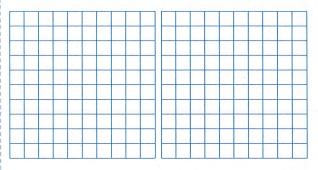
3. Error Analysis Margo is comparing the functions f(x) = |x| and g(x) = |x + 1| - 5. She said the graph of g is a vertical translation of the graph of f 5 units down and a horizontal translation of the graph of f 1 unit right. What is Margo's error?

Do You KNOW HOW?

Graph each function and its parent function.

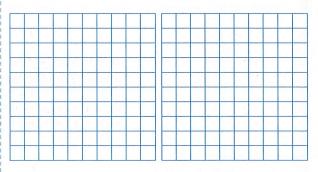
4.
$$g(x) = |x| - 1$$

5.
$$g(x) = (x - 3)^2$$



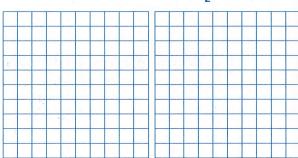
6.
$$g(x) = -|x|$$

7.
$$g(x) = -x$$



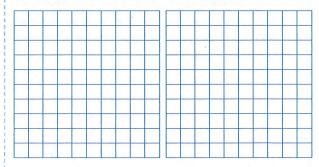
8.
$$g(x) = x^2 - 2$$

9.
$$g(x) = \frac{1}{2}|x|$$



10.
$$g(x) = 4x$$

11.
$$g(x) = |5x|$$



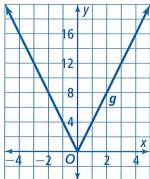
12. Which types of transformations in Exercises 4–11 do not change the shape of a graph? Which types of transformations change the shape of a graph? Explain.



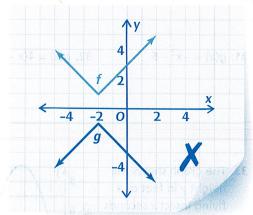


UNDERSTAND

- 13. Use Structure Write a function g with the parent function $f(x) = x^2$ that has a vertex at (3, -6).
- 15. Higher Order Thinking Describe the transformation g of f(x) = |x| as a stretch and as a compression. Then write two equations to represent the function. What can you conclude? Explain.



14. Error Analysis Describe and correct the error a student made in graphing g(x) = f(-x) as a reflection across the y-axis of the graph of f(x) = |x + 2| + 1.



16. Use Structure The graph of the parent function $f(x) = x^2$ is reflected across the y-axis. Write an equation for the function g after the reflection. Show your work. Based on your equation, what happens to the graph? Explain.

17. Error Analysis Monisha is comparing f(x) = |x| and g(x) = |2x - 4|. She said the graph of g is a horizontal translation of the graph of f 4 units to the right and a horizontal compression of the graph of f by a factor of 2. What is Monisha's error?





PRACTICE

Graph each function as a translation of its parent function, f. How did the transformation affect the domain and range? SEE EXAMPLE 1

18.
$$g(x) = |x| - 5$$

19.
$$g(x) = (x+1)^2$$

20.
$$g(x) = |x - 3|$$

21.
$$g(x) = x^2 + 2$$

What is the equation for the image graph? Check by graphing. SEE EXAMPLE 2

22. Reflect
$$f(x) = x^2 + 1$$
 across the x-axis.

23. Reflect
$$f(x) = x^2 + 1$$
 across the y-axis.

Graph each function as a vertical stretch or compression of its parent function.

SEE EXAMPLE 3

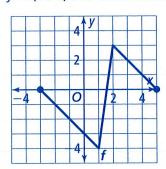
24.
$$g(x) = 0.25|x|$$

25.
$$g(x) = 3x^2$$

26.
$$g(x) = 1.5|x|$$

27.
$$g(x) = 0.75x^2$$

28. Use the graph of f(x) to graph y = f(x + 1) + 2. SEE EXAMPLE 4



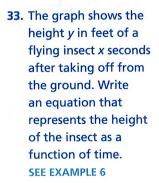
What transformations of $f(x) = x^2$ are applied to the function g? SEE EXAMPLE 5

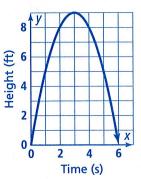
29.
$$g(x) = 2(x+1)^2$$

30.
$$g(x) = (x-3)^2 + 5$$

31.
$$g(x) = -x^2 - 6$$

32.
$$q(x) = 4(x-7)^2 - 9$$



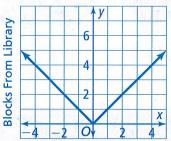






APPLY

34. Model With Mathematics Chiang walks to school each day. She passes the library halfway on her walk to school. She walks at a rate of 1 block per minute. The graph shows the distance Chiang is from the library as she walks to school.



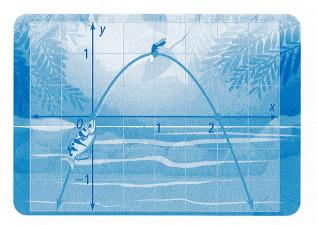
Minutes Before Minutes After **Passing Library Passing Library**

a. Write a function, f, to model the distance Chiang is from the library when she walks to school.

b. If Chiang jogs to school, she travels at a rate of 2.5 blocks per minute. Write a function, g, to model the distance Chiang is from the library when she jogs to school.

c. Graph the function, g, that models the distance Chiang is from the library when she jogs to school.

35. Model With Mathematics The archer fish spits water at flying insects to knock them into the water. The path of the water is shown with x and y distances in feet. Write an equation to represent the path of the water in relation to the coordinate grid. Then determine the coordinates of the point of maximum height of the water.





ASSESSMENT PRACTICE

36. Match each equation on the left to an equation on the right that has the same translation of y = |x|.

A.
$$y = |x| - 1$$

D.
$$y = |x + 1| - 1$$

B.
$$y = -|x + 1|$$

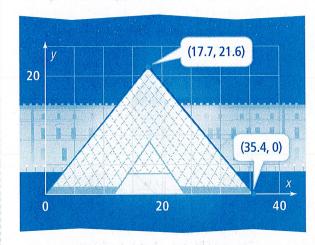
E.
$$y = 2|x| - 1$$

C.
$$y = -|x + 1| - 1$$

F.
$$y = |x + 1|$$

- 37. SAT/ACT Which translation is part of transforming $f(x) = x^2$ into $h(x) = (x + 4)^2 - 2$?
 - A left 4 units
 - ® left 2 units
 - © right 2 units
 - D right 4 units

38. Performance Task The Louvre Pyramid in Paris is shown on the coordinate grid, where x and y are measured in meters and the ground is represented by the x-axis.



Part A The outline of the Pyramid is a transformation of the function f(x) = |x|. Write a function g to model the outline of the Pyramid.

Part B What is the domain and range of the function that models the outline of the Pyramid? What do the domain and range represent?



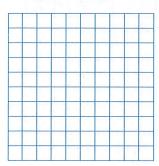
MODEL & DISCUSS

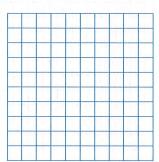
A music teacher needs to buy guitar strings for her class. At store A, the guitar strings cost \$6 each. At store B, the guitar strings are \$20 for a pack of 4.



1-3 Piecewise-Defined **Functions** PearsonRealize.com

A. Make graphs that show the income each store receives if the teacher needs 1-20 guitar strings.





B. Describe the shape of the graph for store A. Describe the shape of the graph for store B. Why are the graphs different?

C. Communicate Precisely Compare the graphs for stores A and B. For what numbers of guitar strings is it cheaper to buy from store B? Explain how you know.

HABITS OF MIND

Communicate Precisely Why do you use dots rather than line segments to graph these two functions?





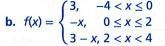
- 1. How much will Alani earn if she works:
 - a. 37 hours?

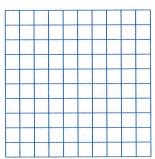
b. 43 hours?

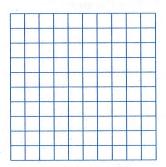


2. Graph the piecewise-defined function. What are the domain and range? Over what intervals is the function increasing or decreasing?

a.
$$f(x) = \begin{cases} 2x + 5, & -6 \le x \le -2 \\ 2x^2 - 7, & -2 < x < 1 \\ -4 - x, & 1 \le x \le 3 \end{cases}$$
 b. $f(x) = \begin{cases} 3, & -4 < x \le 0 \\ -x, & 0 \le x \le 2 \\ 3 - x, & 2 < x < 4 \end{cases}$







HABITS OF MIND

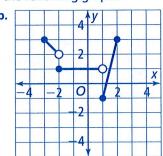
Reason Why is the interval for the domain of the second piece of the function in Try It! 2(a) defined using the < symbol rather than the ≤ symbol?



Try It! Write a Piecewise-Defined Rule From a Graph

3. What rule defines the function in each of the following graphs?

0



- 4. How can you rewrite each function as a piecewise-defined function?
 - a. f(x) = |-5x 10|

b. f(x) = -|x| + 3

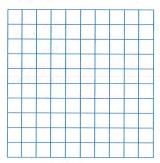
HABITS OF MIND

Use Structure Why can the graph of an absolute value function also be defined as a piecewise-defined function?

EXAMPLE 5 Try It! Graph a Step Function

5. The table below represents fees for a parking lot. Graph the function. What are the domain and range of the function? What are the maximum and minimum values?

Time	$0 < t \le 3h$	$3 < t \le 6h$	$6 < t \le 9h$	$9 < t \le 12h$
Cost	\$10	\$15	\$20	\$25



HABITS OF MIND

Make Sense and Persevere How would a piecewise-defined rule for the function in the Try It! show that the graph is a step function?



Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How do you model a situation in which a function behaves differently over different parts of its domain?

2. Vocabulary How do piecewise-defined functions differ from step functions?

3. Error Analysis Given the function

$$f(x) = \begin{cases} 2x + 5, & -2 < x \le 4 \\ -4x - 7, & 4 < x \le 9 \end{cases}$$

 $f(x) = \begin{cases} 2x + 5, -2 < x \le 4 \\ -4x - 7, 4 < x \le 9 \end{cases}$ Rebecca says there is an open circle at x = 4for both pieces of the function. Explain her

4. Communicate Precisely What steps do you follow when graphing a piecewise-defined function?

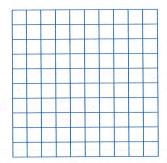
5. Make Sense and Persevere Is the relation defined by the following piecewise rule a function? Explain.

$$y = \begin{cases} 7x - 4, & x < 2 \\ -x + 5, & x \ge -2 \end{cases}$$

Do You KNOW HOW?

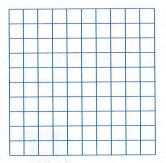
Graph the function.

6.
$$f(x) = \begin{cases} -x + 1, & -10 \le x < -3 \\ x^2 - 9, & -3 \le x \le 3 \\ 2x + 1, & 3 < x < 5 \end{cases}$$



7.
$$g(x) =$$

$$\begin{cases}
1, & 0 \le x < 2 \\
3, & 2 \le x < 4 \\
5, & 4 \le x < 6 \\
7, & 6 \le x < 8
\end{cases}$$

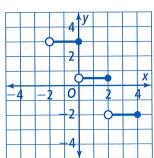


8. Given the function

$$f(x) = \begin{cases} -2x + 4, & 0 \le x < 8 \\ -5x + 11, & x \ge 8 \end{cases}$$

is the function increasing or decreasing over the interval [2, 7]? Find the rate of change over this interval.

9. What is the rule that defines the function shown in the graph?



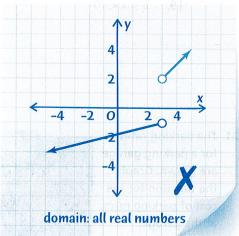




UNDERSTAND

10. Communicate Precisely What do closed circles and open circles on the graph of a step function indicate?

11. Error Analysis What error did Damian make when defining the domain of the graph? Explain.



12. Communicate Precisely For what values of x is the function $f(x) = \begin{cases} -3x + 4, & -2 < x \le 3\\ 2x + 1, & 4 \le x < 9 \end{cases}$

13. Mathematical Connections For the piecewisedefined function f(x) =find two x-values that have the same y-value and the sum of the x-values is 10.

14. Higher Order Thinking The function $f(x) = \lfloor x \rfloor$ is called the greatest integer function because the output returned is the greatest integer less than or equal to x. For example, $f(3.2) = \lfloor 3.2 \rfloor = 3$ and $f(0.975) = \lfloor 0.975 \rfloor = 0$. Graph the function $f(x) = \lfloor x \rfloor$. What type of graph does this look like?





PRACTICE

15. A phone company offers a monthly cellular phone plan for \$25. The plan includes 250 anytime minutes, and charges \$0.20 per minute above 250 min. Write a piecewise-defined function for C(x), the cost for using x minutes in a month. SEE EXAMPLE 1

Write each absolute value function as a piecewise-defined function. SEE EXAMPLE 4

18.
$$f(x) = |3x + 1|$$

19.
$$g(x) = |-2x - 6|$$

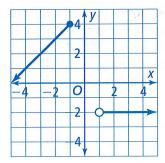
16. Graph the piecewise-defined function. State the domain and range. Identify whether the function is increasing, constant, or decreasing on each interval of the domain. SEE EXAMPLE 2

$$f(x) = \begin{cases} \frac{1}{4}x + 3, & -2 < x \le 0 \\ 2, & 0 < x \le 4 \\ 3 - x, & 4 < x \le 7 \end{cases}$$

Graph the step function. SEE EXAMPLE 5

20.
$$f(x) = \begin{cases} 2, & -3 \le x < 1 \\ 5, & 1 \le x < 4 \\ 8, & 4 \le x < 6 \\ 9, & 6 \le x < 10 \end{cases}$$

17. Write the rule that defines the function in the following graph. SEE EXAMPLE 3



21. The parking rates for a parking garage are shown. Graph the function for the cost of parking rates at the garage. SEE EXAMPLE 5







APPLY

- 22. Model With Mathematics If Kyle works more than 40 h per week, his hourly wage for the extra hour(s) is 1.5 times the normal hourly wage of \$10 per hour. Write a piecewisedefined function that gives Kyle's weekly pay P in terms of the number h of hours he works. Determine how much Kyle will get paid if he works 45 h.
- b. Sarah uses approximately 1,500 texts per month. What is the monthly cost under each text message plan?

23. Model With Mathematics Text message plans offered at a phone company, along with overage charges, are shown.



a. Write a function for each plan where x is the number of texts and f(x) is the total monthly cost.

c. Write an interval for the number of text messages that would make each plan the best one to purchase.

24. Reason The cost C (in dollars) of sending next-day mail depends on the weight x (in ounces) of a package. The cost of packages, up to 5 lb, is given by the function below. What are the domain and range of the function?

$$f(x) = \begin{cases} 12.25, & 0 < x \le 8 \\ 16.75, & 8 < x \le 32 \\ 19.50, & 32 < x \le 48 \\ 23.50, & 48 < x \le 64 \\ 25.25, & 64 < x \le 80 \end{cases}$$

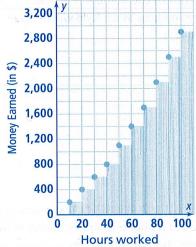


ASSESSMENT PRACTICE

25. Does the function have a range of $(-\infty, 4)$? Write **yes** or **no**.

was fall and the same server by	Yes	No
a. $f(x) = \begin{cases} x-3, & \text{if } x < -2 \\ 5-x, & \text{if } x > 1 \end{cases}$		
b. $h(x) = \begin{cases} x-1, & \text{if } x < -1 \\ 3-x, & \text{if } x > 2 \end{cases}$		
c. $g(x) = \begin{cases} x + 5, & \text{if } x < -1 \\ -x - 5, & \text{if } x > 1 \end{cases}$		
d. $k(x) = \begin{cases} x + 2, & \text{if } x < -2 \\ x + 4, & \text{if } x > 1 \end{cases}$	a	

27. Performance Task Yama works a varying number of hours per month for a construction company. The following scatter plot shows how much money he earns for each number of hours he works. Write the piecewise-defined function that represents Yama's earnings as a function of his hours worked.



26. SAT/ACT What is the vertex of the absolute value function f(x) = -|x - a| + b where a and b are real numbers?

$$^{\circ}$$
 $(-a, b)$

$$\bigcirc$$
 (a, $-b$)

$$\textcircled{D}(-a, -b)$$



CRITIQUE & EXPLAIN

Yumiko and Hugo are looking at the table of data.

Yumiko writes

$$f(1) = 1 + 4 = 5$$

$$f(2) = f(1) + 4 = 5 + 4 = 9$$

$$f(3) = f(2) + 4 = 9 + 4 = 13,$$

$$f(4) = f(3) + 4 = 13 + 4 = 17.$$

Hugo writes g(x) = 1 + 4x.

e next	Input	Output	
	0 101 1	a. 2 1 , 20 1	
	1	5	
	2	9	
	3	13	
	4 80	7 5 17 d	
	4		e de la composição de l

1-4
Arithmetic Sequences and Series

A. Describe the pattern Yumiko found for finding an output value.

B. Describe the pattern Hugo found for finding an output value.

C. Use Structure Compare the two methods. Which method would be more useful in finding the 100th number in the list? Why?

HABITS OF MIND

Use Structure Find the average rate of change between a few pairs of points. What can you conclude about the function represented in the table?



- 1. Are the following sequences arithmetic? If so, what is the recursive definition, and what is the next term in the sequence?
 - a. 25, 20, 15, 10, ...
 - **b.** 2, 4, 7, 12, 13, . . .

EXAMPLE 2 Try It! Translate Between Recursive and Explicit Forms

- **2. a.** For the recursive definition $a_n = \begin{cases} 45, & n = 1 \\ a_{n-1} 2, & n > 1 \end{cases}$ what is the explicit definition?
 - **b.** For the explicit definition $a_n = 1 + 7(n 1)$, what is the recursive definition?

EXAMPLE 3 Try It! Solve Problems With Arithmetic Sequences

- 3. Samantha is training for a race. The distances of her training runs form an arithmetic sequence. She runs 1 mi the first day and 2 mi the seventh day.
 - a. What is the explicit definition for this sequence?
 - b. How far does she run on day 19?

HABITS OF MIND

Use Appropriate Tools How can you use the recursive definition for an arithmetic sequence to find the 120th term?





Try It! Find the Sum of an Arithmetic Series

- 4. Find the sum of each arithmetic series.
 - a. series with 12 terms, $a_1 = 3$ and $a_{12} = 25$

b. 5 + 11 + 17 + 23 + 29 + 35 + 41



Try It! Use Sigma Notation

- **5.** a. What is the sum of the series $\sum_{i=1}^{13} 3i + 2$?
 - **b.** How can you write the series 8 + 13 + 18 + ... + 43 using sigma notation? What is the sum?

EXAMPLE 6



Try It! Use a Finite Arithmetic Series

6. A flight of stairs gets wider as it descends. The top stair is 15 bricks across, the second stair is 17 bricks across, and the third stair is 19 bricks across. What is the total number of bricks used in all 16 stairs?

HABITS OF MIND

Make Sense and Persevere What is the sum of the first 50 odd whole numbers? Explain how you found your answer.



Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION What is an arithmetic sequence, and how do you represent and find its terms and their sums?

2. Vocabulary How do arithmetic sequences differ from arithmetic series?

3. Error Analysis A student claims the sequence 0, 1, 3, 6, . . . is an arithmetic sequence, and the next number is 10. What error did the student make?

4. Communicate Precisely How would you tell someone how to calculate $\sum_{n=1}^{5} (2n+1)$?

Do You KNOW HOW?

Find the common difference and the next three terms of each arithmetic sequence.

5.
$$\frac{1}{4}$$
, $\frac{1}{2}$, $\frac{3}{4}$, 1, $\frac{5}{4}$, ...

11. In June, you start a holiday savings account with a deposit of \$30. You increase each monthly deposit by \$4 until the end of the year. How much money will you have saved by the end of December?



UNDERSTAND

12. Use Structure Write an arithmetic sequence with at least four terms, and describe it using both an explicit and recursive definition.

13. Error Analysis Alex says the common difference for an arithmetic sequence is always negative because of the definition of *difference*. Why is he wrong? Write an arithmetic sequence to show he is wrong.

14. Use Structure A company will pay Becky \$120 for her first sale. For each sale after that, they will pay an extra \$31.50 per sale. So, she will make \$151.50 for the second sale, \$183 for the third sale, and so on. How many sales will Becky have to make to earn at least \$2,000?

- **15. Higher Order Thinking** Felipe and Gregory are given the arithmetic sequence -1, 6, 13, \dots Gregory wrote the explicit definition $a_n = -1 + 7(n-1)$ for the sequence. Felipe wrote the definition as $a_n = 7n 8$. Which one of them is correct? Explain.
- 16. Model With Mathematics Suppose you are building 10 steps with 8 concrete blocks in the top step and 80 blocks in the bottom step. If the number of blocks in each step forms an arithmetic sequence, find the total number of concrete blocks needed to build the steps.
- 17. Model With Mathematics With her half-marathon quickly approaching, Talisa decides to train every day up to the day of the race. She plans to run 2 mi the first day and 3.2 mi the fifth day.



- **a.** What is the explicit definition for this sequence?
- **b.** Which day of training will she run the distance of a half-marathon (13.1 mi)?



PRACTICE

Are the following sequences arithmetic? If so, what is the common difference? What is the next term in the sequence? SEE EXAMPLE 1

Translate between the recursive and explicit definitions for each sequence. SEE EXAMPLE 2

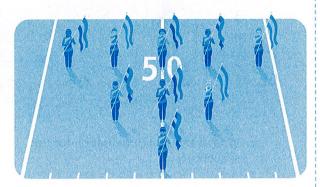
22.
$$a_n = \begin{cases} 2, n = 1 \\ a_{n-1} + 2, n > 1 \end{cases}$$

23.
$$a_n = -2 + 7(n-1)$$
 24. $a_n = \frac{1}{8}(n-1)$

24.
$$a_n = \frac{1}{8}(n-1)$$

25.
$$a_n = \begin{cases} -4, & n = 1 \\ a_{n-1} - 4, & n > 1 \end{cases}$$

26. The members of a school's color guard begin their performance in a pyramid formation. The first row has 1 member, and the third row has 5 members. SEE EXAMPLE 3



a. What is the explicit definition for this sequence?

b. How many members are in the eighth row?

Find the sum of an arithmetic series with the given number of terms, a_1 , and a_n . SEE EXAMPLE 4

27. 10 terms,
$$a_1 = 4$$
, $a_{10} = 31$

28. 15 terms,
$$a_1 = 17$$
, $a_{15} = 129$

What is the sum of each of the following

29.
$$\sum_{n=1}^{11} (3+2n)$$

series? SEE EXAMPLE 5
29.
$$\sum_{n=1}^{11} (3+2n)$$
 30. $\sum_{n=1}^{12} (\frac{n}{2}-9)$

31. The number of seats in each row of an auditorium increases as you go back from the stage. The front row has 24 seats, the second row has 29 seats, and the third row has 34 seats. If there are 35 rows, how many seats are in the auditorium? SEE EXAMPLE 6



APPLY

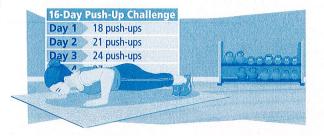
32. Make Sense and Persevere A piece of tile artwork is in the shape of a triangle. The top row has 1 tile, the second row has 2 tiles, and the third row has 3 tiles. If there are 14 rows of tiles, how many tiles were used to make the artwork?

33. Model With Mathematics A race car driver travels 34 ft in the first second of a race. If the driver travels 3.5 additional feet each subsequent second, how many feet did the driver travel in 52 s?

- 34. Construct Arguments A school board committee has decided to spend its annual technology budget this year on 90 student laptops and plans to buy 40 new laptops each year from now on.
 - a. The school board decided that each student in the school should have access to a laptop in the next ten years. If there are 500 students, will the technology coordinator meet this goal? Explain.

b. What are some pros and cons of buying student laptops in this manner? If you could change the plan, would you? If so, how would you change it?

35. Make Sense and Persevere On October 1, Nadia starts a push-up challenge by doing 18 push-ups. On October 2, she does 21 push-ups. On October 3, she does 24 push-ups. She continues until October 16, when she does the final push-ups in the challenge.



a. Write an explicit definition to model the number of push-ups Nadia does each day.

b. Write a recursive definition to model the number of push-ups Nadia does each day.

c. How many push-ups will Nadia do on October 16?

d. What is the total number of push-ups Nadia does from October 1 to October 16?



ASSESSMENT PRACTICE

36. Which of the following are also numbers in the arithmetic sequence 4, 11, 18, 25, 32, . . . ? Write the numbers in the correct box.

60 68 75 39 81

In the sequence Not in the sequence

37. SAT/ACT Tamika is selling magazines door to door. On her first day, she sells 12 magazines, and she intends to sell 5 more magazines per day than on the previous day. If she meets her goal and sells magazines for a total of 10 days, how many magazines would she sell?

A 314

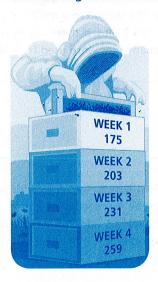
B 345

C 415

D 474

E 505

38. Performance Task The chart shows the population of Edgar's beehive over the first four weeks. Assume the population will continue to grow at the same rate.



Part A Write an explicit definition for the sequence.

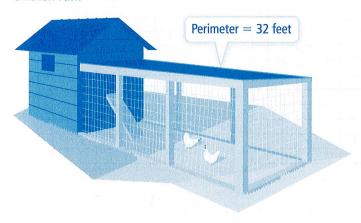
Part B If Edgar's bees have a mass of 1.5 g each, what will the total mass of all his bees be in 12 wk?

Part C When the colony reaches 1,015 bees, Edgar's beehive will not be big enough for all of them. In how many weeks will the bee population be too large?



MODEL & DISCUSS

A homeowner has 32 feet of fencing to build three sides of a rectangular chicken run.

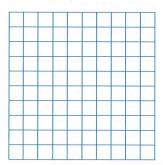


1-5
Solving Equations and Inequalities by Graphing



A. Make a table of values for the length, width, and area of different rectangles that will utilize 32 feet of fencing. Then write a function for the area, in terms of width, of a rectangular run using this much fencing.

B. Graph your function.



C. Reason Explain what happens where the graph intersects the x-axis.

HABITS OF MIND

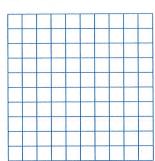
Make Sense and Persevere For what widths will the area of the chicken run be at least 55 ft²?



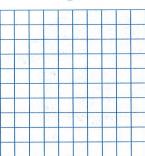
EXAMPLE 1 Try It! Use a Graph to Solve an Equation

1. Use a graph to solve the equation.

a.
$$5x - 12 = 3$$



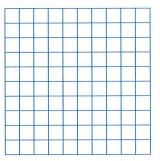
b.
$$-|x-2| = -\frac{1}{2}x - 2$$



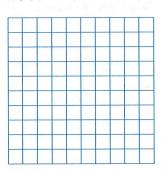
EXAMPLE 2 Try It! Solve a One-Variable Inequality by Graphing

2. Use a graph to solve each inequality.

a.
$$x^2 + 6x + 5 \ge 0$$



b.
$$x + 3 > 7 - 3x$$



HABITS OF MIND

Use Structure How does a graph show the solution to an equation?



EXAMPLE 3 Try It! Use a Table to Solve an Equation

3. The equation $x^2 - 4x + 1 = x - 2$ has a second solution in the interval 4 < x < 5. Use a spreadsheet to approximate this solution to the nearest thousandth.

Try It! Use Graphing Technology to Solve Equations **EXAMPLE 4**

4. Use graphing technology to approximate the solutions of the equation $x^2 + 2x - 1 = |x + 2| + 2$ to the nearest tenth.

HABITS OF MIND

Use Appropriate Tools What are the advantages and disadvantages of using spreadsheets and graphing technology?

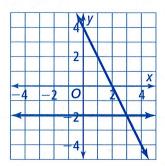


Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How can you solve an equation or inequality by graphing?

Do You KNOW HOW?

4. Using the graph below, what is the solution to -2x + 4 = -2? How can you tell?



2. Communicate Precisely What is an advantage of solving an equation graphically by finding the points of intersection?

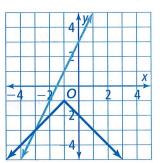
3. Error Analysis Ben said the graph of the inequality $-x^2 + 9 > 0$ shows the solution is x < -3 or x > 3. Is Ben correct? Explain.



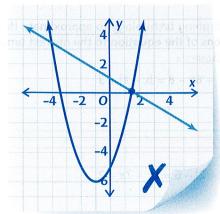


UNDERSTAND

- 5. Construct Arguments Use a graph to solve the equation 3x 5 = 2 + 3x. How can you use algebra to confirm that your graph shows the correct solution?
- 7. Higher Order Thinking Sadie used a graph to solve an equation. What equation did Sadie solve? Explain how to verify your equation is correct.



6. Error Analysis Victor graphed the equation $x^2 + 2x - 5 = -0.6x + 1$. He used the INTERSECT feature on his graphing calculator to find the solution. Victor said one of the solutions is $x \approx 0.116$. Describe and correct the error Victor made.



8. Communicate Precisely Explain how to use the table to find the approximate solution to the equation f(x) = g(x).

X	f(x)	g(x)
1.1426	1.8556	1.857175
1.1427	1.8562	1.8571625
1.1428	1.8568	1.85715
1.1429	1.8574	1.8571375
1.1430	1.858	1.857125



PRACTICE

Use a graph to solve each equation. SEE EXAMPLE 1

9.
$$-x + 4 = 2$$

10.
$$|x-4|-4=\frac{1}{2}x$$

11.
$$3x + 2 = x + 4$$

12.
$$-\frac{1}{4}x + 6 = \frac{1}{2}x + 3$$

13.
$$\frac{3}{4}x = 2x - 10$$

14.
$$|x + 8| = |x - 2|$$

Use a graph to solve each inequality. SEE EXAMPLE 2

15.
$$x^2 - 7x - 8 > 0$$

16.
$$x - 5 > -2x + 4$$

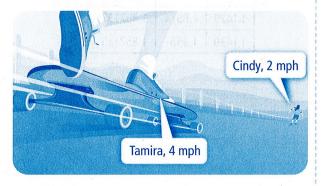
17.
$$x^2 + x - 6 < 0$$

17.
$$x^2 + x - 6 < 0$$
 18. $x^2 + 2x - 8 \le 0$

19.
$$-x^2 - 2x + 15 \le 0$$

19.
$$-x^2 - 2x + 15 \le 0$$
 20. $-x + 5 < \frac{1}{2}x - 1$

21. Cindy is longboarding 6 mi ahead of Tamira. Cindy is traveling at an average rate of 2 mph. Tamira is traveling at a rate of 4 mph. Let x represent the number of hours since Tamira started longboarding. When will Tamira be ahead of Cindy? Write an inequality to represent this situation.



Use a graph and tables to solve the equation.

SEE EXAMPLE 3

22.
$$x^2 - 8x + 5 = x + 3$$

23.
$$\frac{1}{4}x + 3 = x^2 - x + 2$$

24.
$$2x^2 - 5 = -x^2 + 2x - 1$$

25.
$$3x - 4 = \frac{1}{2}|x - 5|$$

Use graphing technology to approximate the solutions of the equation to the nearest tenth.

SEE EXAMPLE 4

26.
$$x^2 + 6x - 8 = |x - 1| + 3$$

27.
$$|x + 2| - 6 = x^2 - 7x - 2$$

28.
$$\frac{1}{5}|x+2|-3=-|x-1|+5$$

29.
$$x^2 + 3x - 7 = -2x^2 - 6x + 9$$



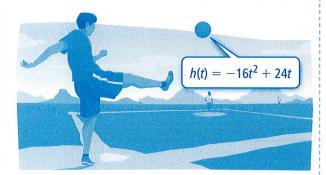


APPLY

30. Reason Jack is running 2.45 mi ahead of Zhang. Jack is running at an average rate of 5.5 mph. Zhang is running at a rate of 7.75 mph. Let x represent the number of hours since Zhang started jogging.

PRACTICE & PROBLEM SOLVING

- a. Write an inequality to represent this situation.
- b. Use graphing technology to find when Zhang will be ahead of Jack. Round to the nearest hundredth.
- 31. Use Structure In a kickball game, a ball is kicked and travels along a parabolic path. The height h, in feet, of the kickball t seconds after the kick can be modeled by the equation $h(t) = -16t^2 + 24t.$



a. A fielder runs a route that will allow him to catch the kickball at about 3 ft above the ground. Write an equation that can be used to find when the fielder will catch the ball.

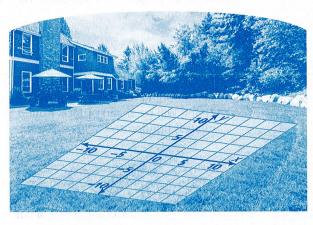
b. Use graphing technology to find out how long the kickball has been in the air when the fielder catches it on its descent. Round to the nearest hundredth.

- 32. Make Sense and Persevere The amount. in millions of dollars, that a company earns in revenue for selling x items, in thousands, is $R = -2x^2 + 18x - 2$. The expenses, in millions of dollars, for selling x items, in thousands, is E = -0.25x + 6.
 - a. The profit P, in millions of dollars, for selling x items, in thousands, is the difference between the revenues and the expenses. Write an inequality that models the company earning a profit.

b. Use graphing technology to find how many items the company must sell to earn a profit. Round to the nearest item.

ASSESSMENT PRACTICE

- **33.** Graph the equation $x^2 3 = x + 3$. What are the solutions to the equation?
- 35. Performance Task Deondra is using a coordinate grid to model her backyard. She wants to mark an area in her backyard to plant a garden. She decides the center of her garden will be located at the origin on her coordinate grid. She models the outline of her garden on the coordinate grid with the inequalities $-\frac{1}{2}|x| + 5 \ge y$ and $y \le \frac{1}{2}|x| 5$.



Part A Graph the inequalities that model the area of Deondra's garden on the coordinate grid.

- **34. SAT/ACT** A graph shows the solution to the equation $-\frac{1}{2}x + \frac{7}{2} = -x + a$ is x = -1. What is the value of a?
 - \bigcirc -3
 - $^{\odot}$ -2
 - © 1
 - D 2
 - **E** 3

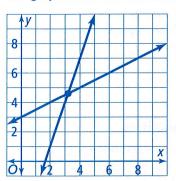
Part B What shape is Deondra's garden?

Part C Deondra wants to cover her garden with garden soil. She wants the soil to be $\frac{1}{2}$ ft deep. If each unit on the coordinate grid represents 1 ft², how much garden soil will Deondra need?



EXPLORE & REASON

The graph shows two lines that intersect at one point.



Linear Systems PearsonRealize.com

A. What are the approximate coordinates of the point of intersection?

B. How could you verify whether the coordinates you estimated are, in fact, the solution? Is the point the solution to the equations of both lines?

C. Make Sense and Persevere Use your result to refine your approximation, and try again. Can you find the point of intersection this way? Is there a more efficient way?

HABITS OF MIND

Communicate Precisely The graphs of two equations appear to intersect at the point (2, 3). Does that guarantee that x = 2 and y = 3 is a solution to both equations? Explain.



EXAMPLE 1



Try It! Solve a System of Linear Equations

1. Solve each system of equations.

a.
$$\begin{cases} 2x + y = -1 \\ 5y - 6x = 7 \end{cases}$$

b.
$$\begin{cases} 3x + 2y = 5 \\ 6x + 4y = 3 \end{cases}$$

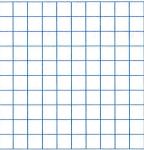
EXAMPLE 2



Try It! Solve a System of Linear Inequalities

2. Sketch the graph of the set of all points that solve this system of linear inequalities.

$$\begin{cases} 2x + y \le 14 \\ x + 2y \le 10 \end{cases}$$
$$\begin{cases} x \ge 0 \\ y \ge 0 \end{cases}$$



HABITS OF MIND

Make Sense and Persevere Is it possible to solve a system of linear inequalities using the same methods you used to solve a system of linear equations?

EXAMPLE 3



Try It! Solve a System of Equations in Three Variables

3. Solve the following systems of equations.

a.
$$\begin{cases} x + y + z = 3 \\ x - y + z = 1 \\ x + y - z = 2 \end{cases}$$

$$\begin{cases} x-y+z=\\ x+y-z=\end{cases}$$

b.
$$\begin{cases} 2x + y - 2z = 3\\ x - 2y + 7z = 12\\ 3x - y + 5z = 10 \end{cases}$$

$$\begin{cases} x - 2y + 7z = 12 \\ - 2y + 7z = 12 \end{cases}$$

$$3x - y + 5z = 10$$

HABITS OF MIND

Generalize What is the goal of the substitution and elimination methods?



4. Write the matrix for the system of equations or the system of equations for the matrix.

$$a. \begin{cases} 3x - y = 4 \\ -2x + 7y = 20 \end{cases}$$

$$\begin{bmatrix} 0 & 2 & 3 & 4 \\ 8 & -1 & -2 & 5 \\ 2 & 0 & 1 & 9 \end{bmatrix}$$

EXAMPLE 5 Try It! Relate Systems of Equations and Matrices

5. a. Write the system of equations described by the augmented matrix. Describe a real-world situation that could be modeled by the system.

b. What would the matrix $\begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 10 \end{bmatrix}$ represent in terms of your real-world situation?

HABITS OF MIND

Communicate Precisely What characteristics must a system of equations have for it to be appropriate to rewrite it in matrix form?



Do You UNDERSTAND?

1.? ESSENTIAL QUESTION How can you find and represent solutions of systems of linear equations and inequalities?

2. Error Analysis Shandra said the solution of the system of equations $\begin{cases} 2x + y = 3 \\ -x + 4y = -6 \end{cases}$ is (-1, 2). Is she correct? Explain.

3. Communicate Precisely Why is a system of linear inequalities often solved graphically?

4. Make Sense and Persevere How does knowing how to solve a system of two equations in two variables help you to solve a system of three equations in three variables?

5. Vocabulary What is the difference between a coefficient matrix and an augmented matrix?

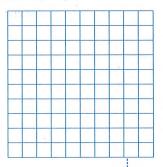
Do You KNOW HOW?

6. Solve the following system of equations.

$$\begin{cases} 2x + 2y = 10\\ x + 5y = 13 \end{cases}$$

7. Graph the following system of inequalities.

$$\begin{cases} -x + 2y < 1\\ x \ge 0\\ y \ge 0 \end{cases}$$



8. Write the system of equations represented by the matrix $\begin{bmatrix} 1 & -2 & /2 \\ -4 & 3 & -5 \end{bmatrix}$.

9. Equations with two variables that are raised only to the first power represent lines. There are three possible outcomes for the intersections of two lines. Describe the outcomes.





UNDERSTAND

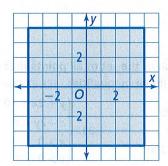
10. Communicate Precisely What is represented by each row in a matrix representing a system of equations?

PRACTICE & PROBLEM SOLVING

13. Use Structure Write a system of equations in three variables with integer solutions. Give the solution. Explain your process.

11. Error Analysis Describe and correct the error a student made in solving the system of equations.

14. Make Sense and Persevere Write a system of inequalities for the shaded region.



- 12. Higher Order Thinking When solving a system of two equations using matrices, what does it mean graphically when the determinant is equal to zero? (Hint: The determinant is (ae - bd) for the coefficient matrix in the form $\begin{bmatrix} a & b \\ d & e \end{bmatrix}$.)
- 15. Mathematical Connections Find a solution to the following system of equations.

$$\begin{cases} x = 5 - 3y \\ y = -2x \end{cases}$$

What is a matrix that could represent the solution that you found?





PRACTICE

Solve the following systems of equations.

SEE EXAMPLE 1

16.
$$\begin{cases} x = 2y - 5 \\ 3x - y = 5 \end{cases}$$

16.
$$\begin{cases} x = 2y - 5 \\ 3x - y = 5 \end{cases}$$
 17.
$$\begin{cases} y = 2x + 3 \\ 2y - x = 12 \end{cases}$$

18.
$$\begin{cases} x - 3y = 1 \\ 2x - y = 7 \end{cases}$$

19.
$$\begin{cases} x + 2y = -4 \\ 3y - y = -5 \end{cases}$$

Sketch the graph of the set of all points that solve each system of linear inequalities. SEE EXAMPLE 2

20.
$$\begin{cases} 0 < x \le 125 \\ x \ge 2y > 0 \\ 2x + 2y \le 300 \end{cases}$$
 21.
$$\begin{cases} y + 2x < 10 \\ x - 2y < 8 \\ x > 0 \end{cases}$$

21.
$$\begin{cases} y + 2x < 10 \\ x - 2y < 8 \\ x > 0 \\ y > 0 \end{cases}$$

Solve the following systems of equations.

SEE EXAMPLE 3

22.
$$\begin{cases} 2x - y - 3z = 20 \\ 3x + y + 6z = 4 \\ x + 2y + 9z = -16 \end{cases}$$
 23.
$$\begin{cases} 2x + 5y - 3z = 14 \\ x - 2y + 4z = -12 \\ -x + 3y - 2z = 13 \end{cases}$$

23.
$$\begin{cases} 2x + 5y - 3z = 14 \\ x - 2y + 4z = -12 \\ -x + 3y - 2z = 13 \end{cases}$$

Write the augmented matrix for each system of equations. SEE EXAMPLE 4

24.
$$\begin{cases} x + y = 2 \\ x - 2y = 17 \end{cases}$$
 25.
$$\begin{cases} y = 2x \\ 4x - y = 9 \end{cases}$$

25.
$$\begin{cases} y = 2x \\ 4x - y = 9 \end{cases}$$

18.
$$\begin{cases} x - 3y = 1 \\ 2x - y = 7 \end{cases}$$
 19.
$$\begin{cases} x + 2y = -4 \\ 3x - y = -5 \end{cases}$$
 26.
$$\begin{cases} 10a - 5b = 3 \\ a = -\frac{1}{2}b \end{cases}$$
 27.
$$\begin{cases} m = 7n - 1 \\ 1 - n = m \end{cases}$$

27.
$$\begin{cases} m = 7n - 1 \\ 1 - n = m \end{cases}$$

Write the system of equations described by each augmented matrix. SEE EXAMPLE 4

28.
$$\begin{bmatrix} 2 & -2 & 4 \\ 1 & 2 & 11 \end{bmatrix}$$

28.
$$\begin{bmatrix} 2 & -2 & 4 \\ 1 & 2 & 11 \end{bmatrix}$$
 29. $\begin{bmatrix} 0.5 & 1 & 0 \\ -1 & 4 & 2 \end{bmatrix}$

30. Charles has a collection of dimes and quarters worth \$1.25. He has 8 coins. What are a system of equations and an augmented matrix that can represent this situation? SEE EXAMPLE 5

31. A set of triangular and square tiles contains 50 pieces and 170 sides. Write a system of equations and an augmented matrix to represent this situation. SEE EXAMPLE 5





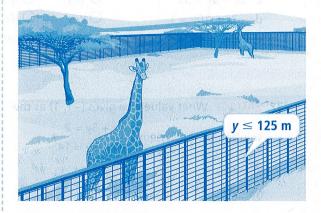
APPLY

32. Model With Mathematics In basketball, a successful free throw is worth 1 point, a basket made from inside the 3-point arc is worth 2 points, and a basket made from outside the 3-point arc basket is worth 3 points. How many of each type of basket did Pilar make?



33. Reason Raul is paid \$75 per week plus \$5 for each new gym membership he sells. He may switch to a gym that pays \$50 per week and \$7.50 for each new membership. How many memberships per week does Raul have to sell for the new gym to be a better deal for him?

34. Reason Keisha is designing a rectangular giraffe enclosure with a length of at most 125 m. The animal sanctuary can afford at most 300 m of fencing, and the length of the enclosure must be at least double the width.



a. Write inequalities to represent each constraint where x = width and y = length.

b. Graph and solve the linear system of inequalities.

c. What does the solution mean?

35. Make Sense and Persevere Ramona needs 10 mL of a 30% saline solution. She has a 50% saline solution and a 25% saline solution. How many milliliters of each solution does she need to create the 30% solution?



ASSESSMENT PRACTICE

36. One equation in a system of equations with one solution is 4x + 2y = 14. Determine if each equation could be the second equation in the system. Select Yes or No.

a. 2x + y = 7

O Yes

represent the situation.

Part A Write a system of equations to

O Yes

O No O No

b. 3x - 6y = -12c. 2x + 6y = 32

O Yes

O No

d. -3x + 10y = 1

- O Yes
- O_{No}

e. 2x + y = 5

- O Yes
- O No
- 37. SAT/ACT What value of a gives (-1, 1) as the solution of the system $\begin{cases} 3x + 5y = 2 \\ ax + 8y = 14 \end{cases}$?

A - 22

- **B** −6
- © 0
- **D** 6
- **E** 22
- 38. Performance Task Each Sophomore, Junior, and Senior at a high school collected aluminum cans and plastic bottles. The table shows the average number of cans and bottles collected per student, by grade level during a 3 week recyling drive.



	Sophomores	Juniors	Seniors
Week 1	3	4	4
Week 2	4	4	3
Week 3	5	6	7

Part B Find the solution of the system of equations you found in Part A.

Part C What does your solution to part B represent in terms of this scenario?



Current Events

You might say that someone who loses their temper has "blown a fuse." However, it's rare to hear about electrical fuses blowing these days. That's because most fuses have been replaced by circuit breakers. A fuse must be replaced once it's blown, but a circuit breaker can be reset.

Ask for permission to look at the electrical panel in your home. If there is a series of switches inside, each of those is a circuit breaker, designed to interrupt the circuit when the electrical current inside is too dangerous. How much electricity does it take to trip a circuit breaker? Think about this question during the Mathematical Modeling in 3-Acts lesson.



ACT 1 **Identify the Problem**

1. What is the first guestion that comes to mind after watching the video?

2. Write down the Main Question you will answer.

3. Make an initial conjecture that answers this Main Question.

4. Explain how you arrived at your conjecture.

5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?



ACT 2 Develop a Model

6. Use the math that you have learned in the topic to refine your conjecture.

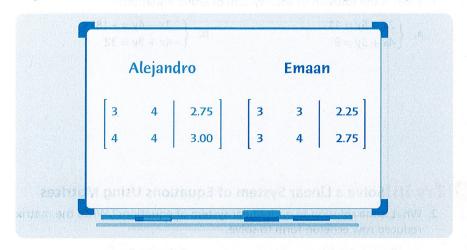
ACT 3 Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?



CRITIQUE & EXPLAIN

Alejandro and Emaan each set up a matrix to represent a system of equations.



A. Write a system of equations that represents each student's matrix.

B. How are Alejandro's and Emaan's matrices alike? How are they different?

C. Use Structure Simplify the equation in each system in which the coefficients for *x* and *y* are the same. How does this change your understanding of the two systems of equations?

HABITS OF MIND

Construct Arguments Write a new matrix by switching the position of the rows in Emaan's matrix. Will the system of equations represented by this new matrix have the same solution as Emaan's original matrix? Explain.





EXAMPLE 1

Try It! Understand Row Operations on a Matrix

1. What is the solution of each system of linear equations?

a.
$$\begin{cases} 2x - 5y = 11 \\ 4x + 3y = 9 \end{cases}$$

b.
$$\begin{cases} 3x - 4y = -18 \\ -4x + 8y = 32 \end{cases}$$

EXAMPLE 2



Try It! Solve a Linear System of Equations Using Matrices

2. What is the solution to each linear system of equations? Write the matrix in reduced row echelon form to solve.

a.
$$\begin{cases} x + y + z = 1 \\ y - z = 3 \\ z = 2x \end{cases}$$

b.
$$\begin{cases} x - y + z = 3 \\ 4x - z = 3 \\ y = -1 \end{cases}$$

HABITS OF MIND

Use Structure What do the zeros in a matrix represent in the system of equations?

EXAMPLE 3



Try It! Use Technology With Matrices

3. What is solution of the linear system of equations?

a.
$$\begin{cases} x + y + z = 55 \\ 2x - y - z = -7 \\ x + 2y - 2z = 10 \end{cases}$$

b.
$$\begin{cases} x + y + z = 1.8 \\ z = 2x + 0.1 \\ 3x + y - z = 0.8 \end{cases}$$





EXAMPLE 4 Try It! Interpret the Reduced Row Echelon Form

4. What is the solution of each system of equations?

a.
$$\begin{cases} x - 2y + z = 8 \\ -2x + 4y - 2z = 16 \\ x + 2y - z = -8 \end{cases}$$

b.
$$\begin{cases} 0.9x - 0.3y + 0.6z = 4.2\\ 2y + 28 = 6x + 4z\\ 3x - y + 2z = 14 \end{cases}$$



EXAMPLE 5 Try It! Apply a Linear System in Three Variables

5. A student has \$128 in a savings account. If she were to withdraw the money and was only given one, five, and twenty-dollar bills, how many bills of each denomination would she have? Assume she has a total of 28 bills, and she has 3 times as many one-dollar bills as she does five-dollar bills.

HABITS OF MIND

Reason How can you tell when a system of equations has no solutions?



Do You UNDERSTAND?

1.9 ESSENTIAL QUESTION How can matrix row operations be used to solve a system of linear equations?

2. Vocabulary The number of rows in a matrix representing a system of equations will be equal to the number of unique variables in the system of equations. True or False? Explain.

3. Error Analysis Dwayne was reducing a matrix into reduced row echelon form. He used row operations to get a matrix with a bottom row that was all zeros. He then added the first row to the bottom row to get a 1 in the bottom row. What error did Dwayne make?

4. Use Appropriate Tools Explain how to use technology to find the solution to a system of equations with three variables.

Do You KNOW HOW?

Solve each system of equations using a matrix.

5.
$$\begin{cases} -x + 2y = -2 \\ x = 6y \end{cases}$$

6.
$$\begin{cases} 2x + 2y = 50 \\ x + y - z = 0 \\ z = 2y - 5 \end{cases}$$

Find the reduced row echelon form of each matrix.

7.
$$\begin{bmatrix} -3 & 2 & 10 \\ 1 & -3 & -22 \end{bmatrix}$$

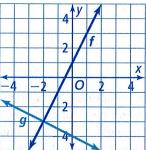
9. Find the reduced row echelon form of $\begin{bmatrix} 0 & -1 \\ 1 & 2 \\ 0 & 3 \end{bmatrix}$.



UNDERSTAND

10. Generalize Is the series of row operations performed on a matrix to get it into reduced row echelon form unique? Is the reduced row echelon matrix unique? Explain.

11. Construct Arguments Would a matrix be useful to calculate the point of intersection of two linear functions f(x) and g(x)? Explain your reasoning.



12. Error Analysis Dylan used a matrix to solve the system of equations below. What error did Dylan make?

$$\begin{cases} 5x + 4y = 2\\ y = x + 5 \end{cases}$$

$$\operatorname{rref} \begin{bmatrix} 5 & 4 & 2\\ 1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -18\\ 0 & 1 & 23 \end{bmatrix}$$

13. Communicate Precisely How is the process of using row operations to transform a matrix into rref form similar to the process of performing operations on equations in order to find a solution?

14. Higher Order Thinking What would be the result of applying row operations to a matrix representing a system of equations for parallel lines?

15. Use Appropriate Tools What characteristics of a system of equations would lead you to use technology to find a reduced row echelon form of a matrix representing the system of equations?

16. Mathematical Connections What is a system of two equations with two unknowns that would result in the following matrix in reduced row echelon form?

$$\begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & -3 \end{bmatrix}$$



PRACTICE

Solve each linear system of equations as a matrix. SEE EXAMPLES 1 AND 2

17.
$$\begin{cases} 2x + 3y = 1 \\ -x = 2y + 1 \end{cases}$$

17.
$$\begin{cases} 2x + 3y = 1 \\ -x = 2y + 1 \end{cases}$$
 18.
$$\begin{cases} \frac{1}{2}x + y = 4 \\ -\frac{1}{4}x - 2y = -5 \end{cases}$$

19.
$$\begin{cases} x + y + z = \\ -y = x \\ 2z + 3y = 0 \end{cases}$$

20.
$$\begin{cases} z = -4x \\ 2x + y = -3 \\ x - y + z = 5.5 \end{cases}$$

Find the reduced row echelon form of each augmented matrix using technology. SEE EXAMPLE 3

21.
$$\begin{bmatrix} 4 & -1 & | & 8 \\ 0 & 2 & | & 16 \end{bmatrix}$$

21.
$$\begin{bmatrix} 4 & -1 & 8 \\ 0 & 2 & 16 \end{bmatrix}$$
 22. $\begin{bmatrix} 0.25 & 4 & 8 \\ 2 & 6 & 12 \end{bmatrix}$

23.
$$\begin{bmatrix} -1 & 1 & -2 & 18 \\ 3 & 0 & -1 & 0 \\ 0 & 6 & 3 & 6 \end{bmatrix}$$
 24.
$$\begin{bmatrix} 1 & 1 & 1 & 17 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix}$$

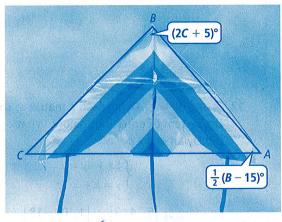
24.
$$\begin{bmatrix} 1 & 1 & 1 & 17 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix}$$

Solve each system of equations using technology with matrices. SEE EXAMPLE 4

25.
$$\begin{cases} 2x + 2y + 2z = 4 \\ -x - y - z = -2 \\ 4z = -4x - 4y + 8 \end{cases}$$

19.
$$\begin{cases} x + y + z = 3 \\ -y = x \\ 2z + 3y = 0 \end{cases}$$
 20.
$$\begin{cases} z = -4x \\ 2x + y = -3 \\ x - y + z = 5.5 \end{cases}$$
 26.
$$\begin{cases} 2x - 2y - 4z = 8 \\ 8x - 8y - 4z = 4 \\ -2x + 2y + 4z = -3 \end{cases}$$

27. Write a matrix to represent the system of equations showing the relationships between angles of the triangle. Then use technology to find the reduced row echelon form of the matrix and identify the measures of each angle. SEE EXAMPLES 5



$$\begin{cases} A + B + C = 180 \\ B = 2C + 5 \\ 2A = B - 15 \end{cases}$$



APPLY

- 28. Reason Talisha receives a \$25 gift card to a digital application store. Each game download costs \$3, and each song download costs \$1. Talisha downloads 1 more song than games and uses all of the \$25. How many of each application did Talisha download?
- 30. Make Sense and Persevere An art supply store orders a total of 80 items of a single color at a time. Colored pencils cost the store \$0.75 each, markers cost \$2.50 each, and acrylic paints cost \$4.00 each. The store budgets \$161.25 per color. The matrix below represents the store manager's information for ordering blue art supplies.



29. Model With Mathematics Noemi, Ines, Deondra, and Carla attend a concert. Noemi purchases 2 posters, 1 shirt, and 1 CD for \$35. Ines purchases 1 shirt, 1 poster, and 2 CDs for \$43. Deondra purchases 2 shirts and 1 poster for \$34. How much will Carla pay if she buys the items shown?



What is the relationship between the number of markers and the number of acrylic paints, based on the third row of the matrix?



ASSESSMENT PRACTICE

31. Complete the table to write matrix A in reduced row echelon form.

$$A = \begin{bmatrix} 2 & 3 & -1 & -6 \\ -1 & 2 & 3 & -5 \\ 3 & 4 & 2 & -4 \end{bmatrix}$$

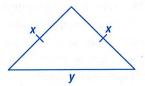
No.			
1	0	0	
0	1	0	1 E (11 (10)
0	0	1	

32. SAT/ACT Which matrix represents the reduced row echelon form of matrix X?

$$X = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -5 & -4 \end{bmatrix}$$

$$\mathbb{B}\begin{bmatrix} 1 & 0 & | -\frac{9}{11} \\ 0 & 1 & | \frac{7}{11} \end{bmatrix}$$

33. Performance Task The triangle has a perimeter of 30 cm, and y is twice the length of x.



Part A Write a system of equations to represent this situation.

Part B Write a matrix to represent the system of equations you wrote in part (a).

Part C Find the reduced row echelon form of the matrix you wrote in part (b).

Part D What does the third column of the matrix you found in part (c) represent?