

Algebra 2 Topic 12: Probability		Estimate Time Frame: 11 Block Days to include Matrices and Statistics
Essential Standards: SP.2, SP.14, SP.15, SP.16, SP.19, N.14		
Assessment Resource: enVision Topic 12 and Formative Assessment Lesson (FAL): Evaluating Statements About Probability		
FCPS Supporting Links		Additional Supporting Links
Pacing Guide enVision Algebra 2 Standards Crosswalk Resource FCPS P-12 Mathematics Guidance Document		Kentucky Academic Standards KSA Blueprint Achieve the Core Operations and Algebraic Thinking Progressions Target of the Standards - conceptual, procedural & application Three-Reads Routine Notice and Wonder Routine MILC Resources Topic 12 Probability <i>enVision Algebra 2 Teacher Guide: page 582A to 582D for specific Topic 12 Focus-Coherence-Rigor</i> MILC Resources Topic 10 Matrices MILC Resources Topic 11 Statistics
Big Ideas		
<p>Students will extend their previous understanding of ratios and basic probability to the probability of multiple events, combinatorics, probability distributions, and expected value.</p> <p>Students will understand and graph probability distributions.</p> <p>Students will use probability models and expected values to make decisions.</p>		
Essential Questions		Common Preconceptions/Misconceptions
<ul style="list-style-type: none"> How can you find the probability of events and 		<ul style="list-style-type: none"> A union of two events, “A or B,” includes all elements in both events

<p>combinations of events?</p> <ul style="list-style-type: none"> •How can you use the language of probability to explain what is meant by a “fair” game? •How would you decide whether two events are mutually exclusive? Are the two events dependent? •How is a permutation different from a combination? 	<p>notated by: $A \cup B$. Addition Rule for mutually exclusive events: If A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. An intersection, “A and B,” of two events includes all overlapping elements notated by: $A \cap B$. A complement for any event A, $P(\text{not } A) = 1 - P(A)$.</p> <ul style="list-style-type: none"> • Students may add probabilities when they should multiply them or multiply them when they should add them. Students can keep two simple examples in mind to help remember the rules. Consider rolling a number cube and finding $P(\text{odd or even})$ as an example of mutually exclusive events. The correct answer is found by adding. Consider flipping two coins and finding $P(\text{heads and heads})$ as an example of independent events. The correct answer is found by multiplying.
Standards for Mathematical Practices	Kentucky Interdisciplinary Literacy Practices (KILP)
<p><u>MP.1. Make sense of problems and persevere in solving them.</u> <u>MP.2. Reason abstractly and quantitatively.</u> <u>MP.3. Construct viable arguments and critique the reasoning of others.</u> <u>MP.4. Model with mathematics.</u> <u>MP.5. Use appropriate tools strategically.</u> <u>MP.6. Attend to precision.</u> <u>MP.7. Look for and make use of structure.</u> <u>MP.8. Look for and express regularity in repeated reasoning.</u></p> <p><i>enVision Teacher Guide: page 582D for specific Math Practice suggestions</i></p>	<ol style="list-style-type: none"> 1. Recognize that text is anything that communicates a message. 2. Employ, develop, and refine schemas to understand and create text. 3. View literacy experiences as transactional, interdisciplinary, and transformational. 4. Utilize receptive and expressive language arts to better understand self, others, and the world. 5. Apply strategic practices, with scaffolding and then independently, to approach new literacy tasks. 6. Collaborate with others to create new meaning. 7. Utilize digital resources to learn and share with others. 8. Engage in specialized, discipline-specific literacy practices. 9. Apply high-level cognitive processes to think deeply and critically about text. 10. Develop a literacy identity that promotes lifelong learning. <p><i>Incorporating texts into math instruction fosters interdisciplinary learning for a more engaging educational experience.</i></p>

Essential Standards	Sample Learning Intentions & Success Criteria	HQIR/Resource Considerations
Cluster: Summarize, represent, and interpret data on a single count or measurement variable.		
<p>KY.HS.SP.2 Use statistics appropriate to describe the shape of the numerical data distribution to compare center (median, mean) and spread (interquartile range when comparing medians and standard deviation when comparing means) of different data distributions. MP.2, MP.6, KILP.1, KILP.2, KILP.7</p> <p><i>Supporting Standard(s): KY.HS.SP.1, KY.HS.SP.10, KY.HS.SP.11</i></p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p>	<p>We are learning to analyze numerical data distributions using appropriate statistics to compare their centers (median, mean) and spreads (interquartile range for comparing medians and standard deviation for comparing means).</p> <ul style="list-style-type: none"> I can determine the appropriate measures of center and spread for a distribution and use them to compare distributions. 	<ul style="list-style-type: none"> Topic 11-1 Topic 11-2 Topic 11-3 <p>Students use raw data and data from appropriate graphical representations to compare differences in the shape, center, spread, and presence of outliers and other unusual features of comparable data sets.</p> <p>Desmos Sampling Strategies: Random Sampling (after 11-2)</p> <p>Desmos Polygraph: Describing Distributions (11-3)</p>
Cluster: Understand independence and conditional probability and use them to interpret data.		
<p>KY.HS.SP.14 Describe events as subsets of a sample space. Use characteristics (or categories) of the outcomes, such as unions, “A or B,” that are mutually exclusive events, unions, “A or B,” that are non-mutually exclusive events, and as intersections, “A and B,” and as complements of other events, “not A,” to calculate basic probabilities. MP.1, MP.2, KILP.1, KILP.6, KILP.7</p>	<p>We are learning to calculate probabilities of multiple events.</p> <ul style="list-style-type: none"> I can define unions, intersections, and complements of events, and calculate probabilities of these events. I can describe events as subsets of a sample space using characteristics of the outcomes (“or,” “and,” “not”). 	<ul style="list-style-type: none"> Topic 12-1 <p>Desmos: Mutually Exclusive (12-1) Desmos: Independence (12-1)</p> <p>3 ACT Math Task (Place</p>

<p><i>Supporting Standard(s): KY.HS.SP.12, KY.HS.SP.13</i></p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p>		<p>Your Guess) (after 12-2)</p> <p>ACT Review SP.14</p>
<p>KY.HS.SP.15 Understand the concept of independence.</p> <p>a. Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities, $P(A) \times P(B)$</p> <p>c. Recognize and explain the concept of independence in everyday language and everyday situations.</p> <p>MP.1, MP.6, KILP.1, KILP.3, KILP.9</p> <p><i>Supporting Standard(s): KY.HS.SP.10</i></p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p>	<p>We are learning to understand the concept of independence.</p> <ul style="list-style-type: none"> I can categorize events as independent or not using the characterization that two events, A and B, are independent when the probability of A and B occurring together is the product of their probabilities. I can determine the outcome of independent events as the product of their probabilities. 	<ul style="list-style-type: none"> Topic 12-1 <p>a. Events A and B are independent if and only if $P(A \text{ and } B) = P(A)P(B)$.</p>
<p>KY.HS.SP.16 Understand the concept of conditional probability.</p> <p>a. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$.</p> <p>c. Recognize and explain the concept of conditional probability in everyday language and everyday situations.</p> <p>d. Find the conditional probability of A given B as the fraction of B's outcomes belonging to A and interpret the answer in terms of the model. MP.1, MP.3, KILP.1, KILP.3, KILP.9</p>	<p>We are learning to understand, recognize, and explain the concept of conditional probability.</p> <ul style="list-style-type: none"> I can understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$. I can interpret the independence of A and B as saying that the conditional probability of A given B is the same as that of A, and that of B given A is the same as that of B. 	<ul style="list-style-type: none"> Topic 12-2 <p>Desmos: Conditional Probability</p>

<p><i>Supporting Standard(s): KY.HS.SP.5</i></p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p>		
<p>Cluster: Use the rules of probability to compute probabilities of compound events</p>		
<p>KY.HS.SP.19 Use permutations and combinations to compute probabilities.</p> <p>a. Distinguish between situations that can be modeled using counting techniques, including the Fundamental Counting Principle, permutations, and combinations.</p> <p>b. Perform calculations using the appropriate counting technique, including simple probabilities.</p> <p>MP.1, MP.8, KILP.1, KILP.2, KILP.7</p> <p><i>Supporting Standard(s): KY.HS.N.5, KY.HS.N.6</i></p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p>	<p>We are learning to use permutations and combinations to compute probabilities of compound events and solve problems.</p> <ul style="list-style-type: none"> I can identify situations that are permutations and those that are combinations. I can use permutations to compute probabilities of compound events and solve problems. I can use combinations to compute probabilities of compound events and solve problems. <p>Number of permutations of n items taken r at a time: ${}_nP_r = \frac{n!}{(n-r)!}$</p> <p>Number of combinations of n items taken r at a time: ${}_nC_r = \frac{n!}{(n-r)!r!}$</p>	<ul style="list-style-type: none"> Topic 12-3 Topic 12-4 Topic 12-6 <p>Permutations are calculated when order matters.</p> <p>Combinations are calculated when order does not matter.</p> <p>Formative Assessment Lesson (FAL): Evaluating Statements About Probability</p>
<p>Cluster: Perform operations on matrices and use matrices in applications.</p>		
<p>KY.HS.N.14 Use matrices to represent and manipulate data. MP.4, MP.5</p> <p><i>Supporting Standard(s): KY.HS.N.5, KY.HS.N.6, KY.HS.N.15</i></p>	<p>We are learning to use matrices to represent and manipulate data, understanding their applications in various contexts.</p> <ul style="list-style-type: none"> I can understand the role of matrices in organizing and representing data in a structured format. 	<ul style="list-style-type: none"> Topic 10-1 Topic 10-2 <p>Students understand that matrices are rectangular</p>

<input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application	<ul style="list-style-type: none"> I can identify situations where matrices can represent data, such as in systems of linear equations, transformations, or statistical analysis. I can represent real-world datasets as matrices, ensuring that the matrices' dimensions match the data. 	<p>arrays comprised of elements that are useful for solving problems in context</p>
Attending to the Standards for Mathematical Practice		
<p>Students encounter chance events in authentic contexts, including situations involving both dependent and independent events, and can determine the difference between the contexts and fluently select and use appropriate formulas (MP.1). Students consider whether the occurrence of one event affects the probability of the other event (MP.2) to determine whether two events are independent. Students analyze and discuss various sources, such as contingency tables, to provide a context for conditional probability (MP.3). They also consider how conditions or assumptions affect probability computation (MP.6).</p>		
Supporting Standards		
<p>KY.HS.N.15 Perform operations with matrices. a. Add, subtract, and multiply matrices of appropriate dimensions. b. Multiply matrices by scalars to produce new matrices. MP.7, MP.8</p> <p>KY.HS.SP.1 Represent the data distribution with plots on the real number line (stem, dot, histograms, and box plots). MP.4, MP.5</p> <p>KY.HS.SP.4(+) When appropriate, fit a normal distribution to a numerical data set for a given mean and standard deviation, and then estimate population percentages using the empirical rule. Recognize that there are data sets for which such a procedure is inappropriate. MP.1, MP.3</p> <p>KY.HS.SP.5 Summarize categorical data for two or more categories in frequency tables. Calculate and interpret joint, marginal, and conditional relative frequencies (probabilities) in the data context, recognizing possible associations and trends in the data. MP.2, MP.7</p> <p>KY.HS.SP.9 Understand statistics as a process for making inferences and justifying conclusions about population parameters based on a random sample from that population. MP.1, MP.3</p> <p>KY.HS.SP.10 Decide if a specified model is consistent with the results from a simulation. MP.3, MP.6</p>		

KY.HS.SP.11 Recognize the purposes and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **MP.3, MP.8**

KY.HS.SP.12 Use data from a sample survey to estimate a population mean or proportion and explain how bias may be involved in the process. **MP.4, MP.7**

KY.HS.SP.13 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between estimates or statistics are significant. **MP.3, MP.8**

KY.HS.SP.15. b (+) Determine whether two events are independent and provide a justification to support the decision.

KY.HS.SP.17 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide whether events are independent and to approximate conditional probabilities. **MP.2, MP.4**

KY.HS.SP.18 (+) Apply the General Multiplication Rule, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, in a uniform probability model, and interpret the answer in terms of the model. **MP.1, MP.2**

KY.HS.SP. 19. c. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

KY.HS.SP.20 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same appropriate graphical displays for data distributions. **MP.3, MP.6,**

KY.HS.SP.21 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution and use the value in analyzing decisions. **MP.1, MP.8**

KY.HS.SP.22 (+) Develop a probability distribution for a random variable.

a. Find an expected value based on a sample space in which theoretical probabilities can be calculated.

b. Find an expected value based on a sample space where empirical probabilities can be calculated. **MP.2, MP.8**

KY.HS.SP.23 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance.

- b. Evaluate and compare strategies based on expected values.
- c. Use calculated expected values to make fair decisions and formulate strategies. **MP.6, MP.8**

Vocabulary

Probability Vocabulary:

Event/Independent events
Outcome
Sample space
Mutually exclusive
Conditional probability
Dependent events
Combination
Factorial
Fundamental counting principle
Permutation
Binomial probability

*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

** Mathematical Practices (A.MP. 1- 8) should be evidenced at some point throughout each unit, depending on the explored tasks. It is important to note that MP. 2 should support learning in every lesson.

*** Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather with other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.