

Algebra 2 Topic 4: Rational Functions		Estimate Time Frame: 11 Block Days
<p>Essential Standards: A.17, F.1.c, F.4, F.8.a, F.8.b</p> <p>Assessment Resource: enVision Topic 4</p>		
FCPS Supporting Links		Additional Supporting Links
<p><a href="#">Pacing Guide</a></p> <p><a href="#">enVision Algebra 2 Standards Crosswalk Resource</a></p> <p><a href="#">FCPS P-12 Mathematics Guidance Document</a></p>		<p><a href="#">Kentucky Academic Standards KSA Blueprint</a></p> <p><a href="#">Achieve the Core Operations and Algebraic Thinking Progressions Target of the Standards</a> - conceptual, procedural &amp; application</p> <p><a href="#">Three-Reads Routine</a></p> <p><a href="#">Notice and Wonder Routine</a></p> <p><a href="#">MILC Resources Topic 4 Rational Functions</a></p> <p><b><i>enVision Algebra 2 Teacher Guide: page 178A to 178D for specific Topic 4 Focus-Coherence-Rigor</i></b></p>
Big Ideas		
<p>Extend their previous understanding of polynomial functions to rational functions.</p> <p>Identify the key features of the graphs of rational functions and learn methods of solving rational equations.</p>		
Essential Questions		Common Preconceptions/Misconceptions
<ul style="list-style-type: none"> <li>• How do you calculate functions defined as quotients of polynomials, and what are the key features of their graphs?</li> <li>• For what input values is the output value positive, negative, or 0?</li> <li>• What happens to the output when the input value gets huge in magnitude?</li> </ul>		<ul style="list-style-type: none"> <li>• Have students use a <i>Think-Aloud</i> strategy to develop the meaning and usage of the phrase “parent function.”</li> <li>• Students should simplify problems and appropriately use previously used vocabulary, such as terms, coefficients, and degrees, as they describe their process.</li> <li>• In addition, students should use prior knowledge to describe the</li> </ul>

	<p>meaning of parts of an expression, such as a particular term or coefficient, and explain the meaning of the entire expression.</p> <ul style="list-style-type: none"> <li>• Use graphs as useful representations for understanding and comparing functions because these behaviors are often easy to see in function graphs. Key features include, but are not limited to, intercepts, intervals where the function is increasing, decreasing, or remaining constant, relative maxima and minima, symmetries, end behavior, and periodicity.</li> <li>• To eliminate misconceptions, plan to approximate solutions with technology.</li> </ul>
Standards for Mathematical Practices	Kentucky Interdisciplinary Literacy Practices (KILP)
<p><a href="#">MP.1. Make sense of problems and persevere in solving them.</a></p> <p><a href="#">MP.2. Reason abstractly and quantitatively.</a></p> <p><a href="#">MP.3. Construct viable arguments and critique the reasoning of others.</a></p> <p><a href="#">MP.4. Model with mathematics.</a></p> <p><a href="#">MP.5. Use appropriate tools strategically.</a></p> <p><a href="#">MP.6. Attend to precision.</a></p> <p><a href="#">MP.7. Look for and make use of structure.</a></p> <p><a href="#">MP.8. Look for and express regularity in repeated reasoning.</a></p> <p><i>enVision Teacher Guide: page 178D for specific Math Practice suggestions</i></p>	<ol style="list-style-type: none"> <li>1. Recognize that text is anything that communicates a message.</li> <li>2. Employ, develop, and refine schemas to understand and create text.</li> <li>3. View literacy experiences as transactional, interdisciplinary, and transformational.</li> <li>4. Utilize receptive and expressive language arts to better understand self, others, and the world.</li> <li><b>5. Apply strategic practices, with scaffolding and then independently, to approach new literacy tasks.</b></li> <li><b>6. Collaborate with others to create new meaning.</b></li> <li><b>7. Utilize digital resources to learn and share with others.</b></li> <li><b>8. Engage in specialized, discipline-specific literacy practices.</b></li> <li><b>9. Apply high-level cognitive processes to think deeply and critically about text.</b></li> <li>10. Develop a literacy identity that promotes lifelong learning.</li> </ol> <p><i>Incorporating texts into math instruction fosters interdisciplinary learning for a more engaging educational experience.</i></p>

Essential Standards	Sample Learning Intentions & Success Criteria	HQIR/Resource Considerations
<b>Cluster: Understand solving equations as a reasoning process and explain the reasoning.</b>		
<p><b>KY.HS.A.17</b> Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise.</p> <p><b>a.</b> Solve rational equations written as proportions in one variable.</p> <p><b>b.</b> Solve radical equations in one variable.</p> <p><b>MP.3, MP.5, MP.7, KILP.1, KILP.2, KILP.6</b></p> <p><i>Supporting Standard(s): KY.HS.A.15</i></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>We are learning to solve radical equations with one variable.</p> <ul style="list-style-type: none"> <li>• I can square both sides of the equation to eliminate the radical and solve for the variable,</li> <li>• I can justify solutions and recognize when extraneous solutions may arise.</li> <li>• I can substitute solutions into the original equation to verify that they satisfy it.</li> </ul>	<ul style="list-style-type: none"> <li>• Topic 4-5</li> </ul> <p><b>3 ACT Math Task (Cool Waters)</b> - (after 4-5)</p> <p>Students analyze solution sets of equations to determine processes (for example, squaring both sides of an equation) that might lead to a solution set that differs from the original equation.</p>
<b>Cluster: Understand the concept of a function and use function notation.</b>		
<p><b>KY.HS.F.1</b> Understand properties and key features of functions and the different ways functions can be represented.</p> <p><b>c.</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the amounts and sketch graphs showing key features given a verbal description of the relationship.</p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>We are learning to graph and interpret key features of graphs of radical functions.</p> <ul style="list-style-type: none"> <li>• I can identify the domain and range of a radical function.</li> <li>• I can sketch the graphs of a radical function.</li> </ul> <p>We are learning to interpret key features of graphs and tables of functions that model relationships between two quantities and sketch the graph.</p> <ul style="list-style-type: none"> <li>• I can sketch graphs that accurately represent the key features described</li> </ul>	<ul style="list-style-type: none"> <li>• Topic 4-1</li> <li>• Topic 4-2</li> </ul> <p>A function is often described and understood in terms of the output behavior, or over what input values it is increasing, decreasing, or constant.</p> <p>Important questions include, "For what input values is the output value positive,</p>

	<p>verbally, including shape, direction, and position of essential points.</p> <ul style="list-style-type: none"> <li>I can interpret key features of graphs and tables given a verbal description.</li> </ul>	<p>negative, or 0? What happens to the output when the input value gets huge in magnitude?"</p>
<b>Cluster: Analyze functions using different representations.</b>		
<p><b>KY.HS.F.4</b> Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★</p> <p><b>b.</b> Graph square root, cube root, and absolute value functions.</p> <p><b>d.</b> Graph exponential and logarithmic functions, showing intercepts and end behavior.</p> <p><b>e.</b> (+) Graph trigonometric functions, showing period, midline, and amplitude.</p> <p><b>f.</b> (+) Graph piecewise functions, including step functions.</p> <p><b>g.</b> (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior</p> <p><b>MP.4, MP.5, MP.8, KILP.1, KILP.7</b></p> <p><i>Supporting Standard(s): KY.HS.F.9, KY.HS.F.12, KY.HS.F.13, KY.HS.F.14</i></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>We are learning to graph exponential functions, demonstrating an understanding of intercepts and end behavior.</p> <ul style="list-style-type: none"> <li>I can identify the end behavior and key features of polynomial functions.</li> <li>I can apply their understanding of exponential functions to solve problems in various contexts, such as population growth, compound interest, and radioactive decay.</li> </ul>	<ul style="list-style-type: none"> <li>Topic 4-1</li> <li>Topic 4-2</li> <li>Topic 4-3</li> </ul>
<b>Cluster: Build new functions from existing functions.</b>		

**KY.HS.F.8** Understand the effects of transformations on the graph of a rational function. **MP.3, MP.5**

a. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); **and** find the value of  $k$ , given the graphs.

b. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

We are learning to understand the effects of transformation on graphs.

- I can identify the value of  $k$ , given a graph.
- I can apply their understanding of functions using technology.

- Topic 4-1

Mastery of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.

### Attending to the Standards for Mathematical Practice

Students use technology to explore how changing the value of  $k$  impacts the function's graph (MP.5).

Students use graphical representations to create plausible arguments about the effects of transformations instead of relying on computational rules (MP.3).

### Supporting Standards

**KY.HS. F. 5. b.** Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. **MP.3, MP.6**

**KY.HS.F.9** Find inverse functions.

a. Given the equation of an invertible function, find the inverse.

b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2, MP.6**

**KY.HS.F.12** Construct **exponential functions**, including **geometric sequences**, given a graph, given a description of a relationship, or two input-output pairs (including reading these from a table). **MP.7, MP.8,**

**KY.HS.F.13** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **MP.7, MP.8**

**KY.HS.F.14** Interpret the parameters in an exponential function in terms of a context. **MP.1, MP.2**

**KY.HS.A.10 (+)** Rewrite simple rational expressions in different forms. **MP.7, MP.8**

Advanced A2 students observe how to write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$  and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ .

Methods of rewriting rational expressions could include, but are not limited to

- Inspection
- Synthetic division
- Long division
- Use of technology

**KY.HS.A.11 (+)** Add, subtract, multiply, and divide rational algebraic expressions. **MP.2, MP.3**

Advanced A2 students go beyond demonstrating procedural fluency and apply this standard in various contextual situations.

**KY.HS.A.12** Create equations and inequalities in one variable and use them to solve problems. **MP.1, MP.4**

**KY.HS.A.13** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **MP.2, MP.5**

**KY.HS.A.15** Rearrange formulas to solve a literal equation, highlighting a quantity of interest, using the same reasoning as in solving equations. **MP.2, MP.7**

**KY.HS.A.16** Understand each step in solving a simple equation, following the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **MP.1, MP.3**

**Vocabulary**

Rational expression  
Simplified form of a rational expression  
Domain  
Real numbers  
Greatest common factor  
Rational equation  
Rational number

## Rational function

\*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

\*\* Mathematical Practices (A.MP. 1- 8) should be evidenced at some point throughout each unit, depending on the explored tasks. It is important to note that MP. 2 should support learning in every lesson.

\*\*\* Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather with other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.