

Unit 4 Title	Estimated Time Frame
Probability and Geometry	35 days
Big Idea	
<p>Investigate chance processes and develop, use, and evaluate probability models. Draw, construct and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</p>	
Essential Question(s)	
<p>How can a model help me solve a probability problem? How can you investigate chance processes and develop, use and evaluate probability models? How can geometry be used to solve problems? How is the circumference of a circle used to derive the area of a circle? How are area and volume properties related?</p>	
<p>Standards for Mathematical Practice (MP.) - The practice standards in bold describe expertise to be intentionally developed in this unit.</p>	<p>Kentucky Interdisciplinary Literacy Practices (KILP.) - The practice standards in bold describe expertise to be intentionally developed in Mathematics.</p>
<p>MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.</p>	<p>KILP.1 Recognize that text is anything that communicates a message. KLIP.2 Employ, develop, and refine schema to understand and create text. KILP.3 View literacy experiences as transactional, interdisciplinary and transformational. KILP.4 Utilize receptive & expressive language arts to better understand self, others, and the world. KILP.5 Apply strategic practices, with scaffolding & then independently, to approach new literacy tasks. KILP.6 Collaborate with others to create new meaning. KILP.7 Utilize digital resources to learn and share with others. KILP.8 Engage in specialized, discipline specific literacy practices. KILP.9 Apply high level cognitive processes to think deeply and critically about text. KILP.10. Develop a literacy identity that promotes lifelong learning.</p>
Common Preconceptions/Misconceptions	
<p>Student thinking about theoretical probability is extended to developing a model (MP.4) that lends structure (MP.7) to an otherwise abstract idea. Students may use this model to explain why a penny comes up heads half the time and tails the other half; however, in an experiment where this event is repeated multiple times, the experimental probability may not be exactly $\frac{1}{2}$ and $\frac{1}{2}$. (MP.8).</p>	

Compound probability may be more difficult for students to understand; tree diagrams, lists, etc., may help students understand the concept. Difficult-to-understand compound events may necessitate a simulation tool, for example, a random digit generator.

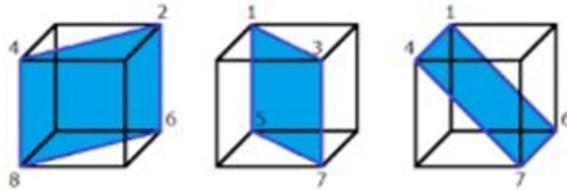
Initially, students may struggle with moving from a concrete understanding of a real-world situation to a miniature version, or vice versa; hands-on measurements and the use of technology can assist students with this abstract idea.

Students may confuse vocabulary words introduced in 7.G.5 (*supplementary, complementary, vertical, and adjacent*). Having students make a foldable where they can make the correct distinction can be helpful.

Students may mischaracterize the volume and surface area of three-dimensional shapes, leading them to develop ways to decide whether a situation calls for the volume of a figure, or the surface area of a figure. Using nets and other appropriate tools gives students a structure to foster a greater understanding of the surface area.

KAS Standards	Prerequisite Skill, Considerations, and Coherence	Samples of Learning Intentions and Success Criteria
<p>KY.7.SP.5 Describe the probability of a chance event as a number between 0 and 1, which tells how likely the event is, from impossible (0) to certain (1). A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. MP.5, MP.6, MP.7</p>	<p>Considerations: Emphasis is on the descriptive language used to describe numerical probabilities; impossible event, unlikely event, equally likely event, likely event, and certain event. Students understand all probabilities must fall between 0 and 1.</p>	<p>We are learning to understand likelihood and probability. (Lesson 7-1)</p> <ul style="list-style-type: none"> ● I can use a number between 0 and 1 to describe the likelihood that an event will occur. ● I can use a word, impossible, unlikely, equally likely, likely, or certain to describe the likelihood that an event will occur. ● I can calculate the probability that an event will occur.
<p>KY.7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the approximate relative frequency given the probability. MP.1, MP.2</p>	<p>Considerations: Estimate the likelihood of an event, test the estimate by trial, and collect data. Students observe the accuracy of the estimate will increase with the frequency of repeated trials.</p>	<p>We are learning to use the experimental probability of an event to solve real-world problems. (Lessons 7-2 , 7-3)</p> <ul style="list-style-type: none"> ● I can find the theoretical probability of an event. ● I can find the experimental probability of an event. ● I can compare the experimental and theoretical probabilities of an event. ● I can use experimental probability to make predictions. ● I can set up a proportion to make a prediction using probability.

<p>KY.7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p>a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events.</p> <p>b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.</p> <p>MP.4, MP.7, MP.8</p>	<p>Considerations:</p> <p>a. If a student is selected randomly from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</p> <p>b. Find the approximate probability a spinning penny will land heads up, or a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</p> <p>Prerequisite Skills: KY.7.RP.3</p>	<p>We are learning to develop a probability model and use it to find probabilities of events then compare the model to observed frequencies..</p> <p>(Lessons 7-3 , 7-4)</p> <ul style="list-style-type: none"> I can determine the total number of possible outcomes for an event and use it to find the probability of a particular outcome. I can create a probability model with a sample space and list of events. I can use the sample space and list of events to estimate the probability of an event. 																		
<p>KY.7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p> <p>a. Explain just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p>b. Represent sample spaces for compound events described in everyday language using organized lists, tables, and tree diagrams.</p> <p>c. Design and use a simulation to generate frequencies for compound events.</p> <p>MP.2, MP.4, MP.7</p>	<p>Considerations:</p> <p>a. If the probability of heads occurring on a coin is $\frac{1}{2}$, then the probability of three heads in a row is $\frac{1}{2} \frac{1}{2} \frac{1}{2} = \frac{1}{8}$</p> <p>b. For a simulation of tossing two fair coins:</p> <div data-bbox="747 870 1360 1089" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center;">Different representation of a sample space</p> <div style="display: flex; align-items: center;"> <table style="margin-right: 20px;"> <tr><td>HH</td><td></td><td></td></tr> <tr><td>HT</td><td>H</td><td>T</td></tr> <tr><td>TH</td><td>H</td><td>T</td></tr> <tr><td>TT</td><td>T</td><td></td></tr> </table> <div style="margin-right: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>H</td><td>T</td></tr> <tr><td>HH</td><td>HT</td></tr> <tr><td>TH</td><td>TT</td></tr> </table> </div> <div> <pre> graph TD Root(()) --- H((H)) Root --- T((T)) H --- HH((HH)) H --- HT((HT)) T --- TH((TH)) T --- TT((TT)) </pre> </div> </div> <p style="font-size: small;">All the possible outcomes of the toss of two coins can be represented as an organized list, table, or tree diagram. The sample space becomes a probability model when a probability for each simple event is specified.</p> </div>	HH			HT	H	T	TH	H	T	TT	T		H	T	HH	HT	TH	TT	<p>We are learning how to determine the outcomes of compound events. (Lesson 7-5)</p> <ul style="list-style-type: none"> I can write the compound probability as a number between 0 and 1. <p>We are learning about probabilities of compound events. (Lessons 7-6, 7-7)</p> <ul style="list-style-type: none"> I can simulate a compound event to generate a sample. I can use a tree diagram, a table, or an organized list to represent the sample space for a compound event. I can find the probability of a compound event using sample space models.
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<p>KY.7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. MP.1, MP.2, MP.5</p>	<p>Considerations: Emphasize converting values from one given measurement to another based on a given scale factor. For example, 1 inch on the scale drawing equals how many feet in real life based on the scale factor given. Students reproduce a given drawing based on a scale factor.</p> <p>Coherence KY.6.G.1→KY.7.G.1→KY.8.EE.6</p>	<p>We are learning to use scale drawings of geometric figures. (Lesson 8-1)</p> <ul style="list-style-type: none"> I can find actual lengths using a scale drawing. I can use scale factors to solve area problems. I can convert a scale drawing to a different scale. 																		

<p>KY.7.G.2 Draw (freehand, with ruler and protractor and with technology) geometric shapes with given conditions. MP.6, MP.7</p>	<p>Considerations: Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p> <p>Coherence KY.7.G.2→KY.8.G.1</p>	<p>We are learning to construct geometric figures. (Lesson 8-2, 8-3)</p> <ul style="list-style-type: none"> I can draw quadrilaterals and triangles using a ruler, protractor, or technology given certain conditions. I can determine the number of triangles that can be formed given side lengths and angle measures.
<p>KY.7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. MP.5, MP.6</p>	<p>Considerations: Cross sections may be taken from horizontal, vertical and oblique angles, such a</p> 	<p>We are learning to describe cross-sections of three-dimensional figures. (Lesson 8-7)</p> <ul style="list-style-type: none"> I can describe and sketch the shape formed when slicing a right rectangular prism or pyramid horizontally, vertically, or diagonally (cross-section).
<p>KY.7.G.4 Use formulas for the area and circumference of circles and their relationships.</p> <p>a. Apply the formulas for the area and circumference of a circle to solve real-world and mathematical problems.</p> <p>b. Explore and understand the relationship between a circle's radius, diameter, circumference, and area. MP.1, MP.2, MP.8</p>	<p>Considerations: Circle Formulas: $C=d\pi$ $C = 2r\pi$ $A=\pi r^2$ Note: Calculating the radius or diameter of a circle given its area is not expected, as finding the square root of a number is reserved for 8th grade.</p> <p>a. Both area and circumference are represented; students recognize when the circumference is needed and when the area is needed.</p> <p>b. Emphasis is on calculating the area given diameter, finding the circumference given radius or diameter, and finding the radius or diameter given circumference. Special attention is given to the relationship between diameter and circumference as a ratio that leads to pi.</p> <p>Coherence KY.7.G.4 → KY.8.G.9</p>	<p>We are learning to solve problems involving the circumference of a circle. (Lesson 8-5)</p> <ul style="list-style-type: none"> I can use the formula <i>Circumference</i> = πd or $2\pi r$ to calculate the circumference of a circle. I can determine the radius or diameter using the circumference formula. <p>We are learning to solve problems involving the area of a circle. (Lesson 8-6)</p> <ul style="list-style-type: none"> I can use the formula <i>Area</i> = πr^2 to calculate a circle's area or find the radius or diameter. I can identify the relationship between the circumference and diameter of a circle.

<p>KY.7.G.5 Apply supplementary, complementary, vertical, and adjacent angles properties in a multi-step problem to write and solve simple equations for an unknown angle in a figure. MP.3, MP.6, MP.7</p>	<p>Considerations: Emphasis is on the relationships between the various angles listed to find missing angles based on the relationships and to write and solve equations to find unknown angles.</p> <p>Coherence KY.4.MD.7 →KY.7.G.5 →KY.8.G.5</p>	<p>We are learning to use angle relationships to solve problems (Lesson 8-4)</p> <ul style="list-style-type: none"> • I can apply properties of supplementary, complementary, vertical, and adjacent angles to find the measures of missing angles. • I can apply the properties of angles to write and solve equations. • I can recognize the relationship between angles formed by intersecting lines and rays. • I can use inverse operations to solve an equation. • I can substitute a variable value in to find the angle measure.
<p>KY.7.G.6 Solve problems involving the area of two-dimensional objects and the surface area and volume of three-dimensional objects.</p> <p>a. Solve real-world and mathematical problems involving the area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons.</p> <p>b. Solve real-world and mathematical problems involving volume and surface area, using nets of three-dimensional objects, including cubes, pyramids, and right prisms. MP.3, MP.4, MP.5</p>	<p>Emphasis is on finding the area of composite figures composed of convex polygons. b. Students understand volume and surface area are two different quantities used to describe the same three-dimensional figure. Building upon their understanding of area, students use nets of three dimensional objects to conceptualize surface area. Students calculate with appropriate units, using nets as a possible strategy for calculation as well as formulas for volume and surface area, where appropriate.</p> <p>Coherence KY.6.G.4→ KY.7.G.6 → KY.8.G.6</p>	<p>We are learning to solve problems involving the area of two-dimensional figures and surface area. (Lesson 8-8)</p> <ul style="list-style-type: none"> • I can find the area of composite 2-dimensional figures by finding the sum of the areas of each part. • I can find the surface area of a 3-dimensional figure by finding the sum of areas of each part of its net. <p>We are learning to solve problems involving the volume of three-dimensional objects. (Lesson 8-9)</p> <ul style="list-style-type: none"> • I can use the area of the base and height of a 3-dimensional figure to find its volume. • I can find the missing dimension when given the volume of a figure.
<p>Supporting Standards</p>		
<p>Students continue their work from grade 6, from solving area problems involving triangles and rectangles to those involving more complex shapes, such as rhombi and trapezoids.</p>		
<p>Essential Vocabulary</p>		

chance event - Anything that happens suddenly or by chance without an apparent cause, ex: Winning the lottery.

combination - A selection in which order is not essential.

compound event - An event whose probability of occurrence depends upon the probability of occurrence of two or more independent events. An event that consists of two or more events that are not mutually exclusive.

permutation - A way to arrange things in which order is important.

probability - A number between 0 and 1 is used to quantify the likelihood of processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

probability distribution – The set of possible values of a random variable with a probability assigned to each.

probability model - A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

random sample - A sample in which every element in the population has an equal chance of being selected.

theoretical probability - The probability/likelihood of an event happening based upon mathematical calculations: $P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{total number of possible outcomes}}$.

adjacent angles - Two angles that share both a side and a vertex.

angle - The union of two rays with a common endpoint, called the vertex.

area - The number of square units that covers a shape or figure.

circle - A closed curve with all its points in one plane and the same distance from a fixed point, the center.

circumference - Distance around a circle; its perimeter.

complementary - Two angles whose sum is 90 degrees.

congruence - Two plane or solid figures are congruent if they have the same size and shape.

cross-section - The intersection of a 3-dimensional body with a plane.

cubes - A solid figure with six square faces; EX: *a die*.

cylinder - A 3-D figure having two parallel bases that are congruent circles; EX: *a tube*.

polygon - A closed plane figure made up of line segments that are joined together.

quadrilateral - A polygon with four sides.

radius - A line segment drawn from the center of a circle to any point on a circle; half the diameter.

right prisms - A prism with bases aligned directly above the other and lateral faces that are rectangles.

right rectangular prism - A 3-D figure with two congruent parallel bases and parallelogram sides; EX: *a box*.

right rectangular pyramid - A polyhedron with a rectangular base and all the other faces triangles meeting at a common vertex at the top known as the apex. The pyramid's apex (top) is directly above the center of the rectangular base.

scale drawing - A drawing that is a reduction or enlargement of the original.

scale factor - A number that multiplies some quantity; the ratio of any two corresponding lengths in two similar geometric figures.

supplementary - Two angles are supplementary if their sum is 180 degrees.

surface area - For a three-dimensional figure, the sum of the areas of all the faces.

two-dimensional - Having the dimensions of length and width.

Common Assessment

Common Assessment Unit 4 Grade 7 Math

Anchor Resources

<i>enVision</i> Topic 7 - Probability	<i>enVision</i> Topic 8 - Geometry
<p>MILC - MILC Probability resources FAL (one per semester) - Evaluating Statements About Probability *** Required 2nd Semester FAL *** <i>This card sort is WORTH the time to use and teach!</i></p> <p>3-ACT Math Task: Photo Finish</p> <ul style="list-style-type: none"> Note: PLAY GAMES! Use enVision Math Online Tools Desmos, <i>What Are My Chances</i> book, MILC probability unit 	<p>MILC - MILC Geometry resources</p> <p>3-ACT-MATH TASK: Whole Lotta Dough</p> <ul style="list-style-type: none"> Mini 1: Scale Drawings and Angles (8-1 to 8-4) combine 8-2/8-3 Mini 2: Circles (8-5 to 8-6 and 3 Act) Mini 3: 3-D Volume, less time on SurfaceArea with nets and cross-sections (8-7 to 8-9) <p>Review conceptual understanding of finding area and volume 6.G.1 and 6.G.2</p>

*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

** Mathematical Practices (A.MP.1- 8) should be evidenced at some point throughout each unit.

*** Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.