6-3 Exponential Growth and Decay NOTES

LEARNING OBJECTIVE: I will learn how to write and solve exponential growth and decay equations.

Exponential Function: $f(x)=a(b)^x$

initial amt, ↵ ↳ common/constant ratio — Growth factor

Exponential Growth (increasing)

$f(t) = a(1 + r)^t$

a = initial amount
r = rate as decimal
t = time

12% → 0.12
6% → 0.06

The population of Hillville grows at 15%. If its current population is 5,000, what will its estimated population be in 5 years?

a = 5000
r = 15% = 0.15
t = 5 yrs

$f(t) = a(1+r)^t$
$f(t) = 5000(1.15)^t$ → function
$f(5) = 5000(1.15)^5$
$f(5) = 10,056$

Desmos → math tools → scientific calc

1 + r = 1 + 0.15

The value of a painting increases by 96% each year. If the painting was purchased four years ago for $9700, how much is it worth today?

a = 9700
r = 96% = 0.96
t = 4

$f(t) = a(1+r)^t$
$f(t) = 9700(1.96)^t$
$f(4) = 9700(1.96)^4$
$f(4) = \$143,151.54$

1 + r
1 + 0.96

Exponential Decay decreasing

$f(t) = a(1 - r)^t$

The population of Central City is currently 300,000. However, it is decreasing by 5% each year. What will its estimated population be in 7 years?

a = 300,000
r = 5% = 0.05
t = 7 yrs

$f(t) = a(1-r)^t$
$f(t) = 300000(0.95)^t$
$f(7) = 300000(0.95)^7$
$f(7) =$

The value of Taran's car depreciates by 10% each year. If his car is currently valued at $10,000, what will its estimated value be in 3 years?

a = 10000
r = 10% = 0.10
t = 3 yrs

$f(t) = a(1-r)^t$
$f(t) = 10000(0.9)^t$
$f(3) = 10000(0.9)^3$

smaller

Interest can be calculated in two ways: Simple Interest, which is paid only on the principal, and Compound Interest, which is paid on both the principal and interest already earned.

Simple Interest: $I = Prt$

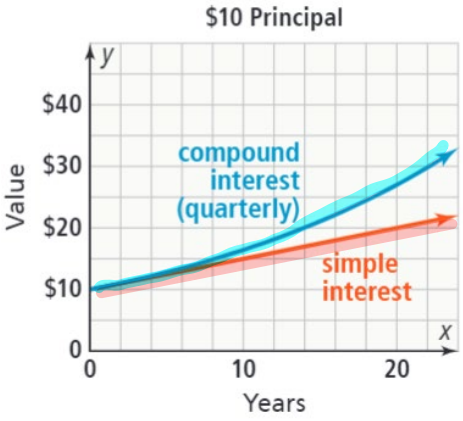
I = Interest
P = Principal
r = rate (as decimal)
t = time

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = Total Amount
P = Principal ~~initial amt.~~
r = rate (as decimal)
n = number times interest is compounded
t = time

annually → n = 1
semi-annually → n = 2
quarterly → n = 4
monthly → n = 12
daily → n = 365

This graph shows $10 at 5% simple interest and 5% compounded quarterly.



* compound interest grows more quickly is exponential

* Simple Interest is linear

This "A" and the "a" from page 1 are NOT the same!

Jan's family invested $3000 for her in a Certificate of Deposit (CD) when she was born. It earns 8% interest compounded quarterly. What is the value after 5 years?

A = ?
P = 3000
r = 8% = 0.08
n = 4
t = 5

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 3000\left(1 + \frac{0.08}{4}\right)^{4(5)}$

$= \$4457.84$

Will the value of Jan's CD be greater after 15 years if it is compounded annually rather than quarterly?

Annually n = 1

$A = 3000\left(1 + \frac{0.08}{1}\right)^{1(15)}$

~~$= \$4447.96$~~

$\$9516.51$

Quarterly n = 4

$A = 3000\left(1 + \frac{0.08}{4}\right)^{4(15)}$

$A = \$9843.09$

The value will be greater if compounded quarterly.

Amy wants to buy a new car that costs $15,600. If the interest is 3% compounded monthly, how much will Amy actually end up paying for her car? * Typical car loan = 5 years

A = ?
P = 15600
r = 3% = 0.03
n = 12
t = 5

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 15600\left(1 + \frac{0.03}{12}\right)^{12(5)}$

$A = \$18,121.22$

Sidney invests $5,000. Her investment earns 6% interest compounded semi-annually. How much will her investment be worth in 10 years?

A = ?
P = 5000
r = 6% = 0.06
n = 2
t = 10

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{0.06}{2}\right)^{2(10)}$

$A = \$9030.56$