Figure out the digits Problem

In the puzzle below, each letter q through z represents a different digit from 0 to 9. Find the correspondence between the letters and the digits. Explain how you found each digit.

1. $u \times r = z$
2. $t + w = t$
3. $r + r + r + r = z$
4. $x + y = q$
5. $\frac{y}{z} = \frac{x}{u}$
6. $s \times v = s$
7. $x^2 = q$
8. $r + r = u$
9. $x + u = s$

Do you need all of the provided information to solve the puzzle? If some of the information is redundant, specify it.

Hints

The following hints direct your attention to various pieces of given information that will help you to identify the digits.

Look through all 9 given expressions. Are there any that immediately point to certain correspondencies between numbers and digits?

What does expression 2 indicate about the value of w?

What does expression 6 indicate about the value of v?

How many choices for q does expression 7 indicate?

Look at expressions 1 and 3 simultaneously. What do they tell you about the value of u?

After you figured out u, look at expression 1 and think which values can z take?

Answers

$q = 9$

$r = 2$

$s = 7$

$t = 5$

$u = 4$

$v = 1$

$w = 0$

$x = 3$

$y = 6$

$z = 8$

No, all provided information is not needed. For example expression 5 is not necessary.

Solutions

In order to solve this problem, you should go through a number of logical conclusions in some order. Below is one possible sequence of steps.

1. From expression 2, $t+w=t$, you may conclude that $w=0$.
2. From expression 6, $s \times v = s$, you may conclude that $v=1$.
3. Fromexpression 7, $x^2 = q$, you may conclude that $x=2$ or $x=3$. The value of x can not equal 1, because then its square would also be x. Also, we've already found that $v=1$. The value of x can not be greater than 3, because in that case its square should be represented by more than 1 digit. If $x=2$, then $q=4$. If $x=3$, then $q=9$.
4. From expression 4, $x+y=q$, you may conclude that $x \neq 2$. If x was equal to 2, then $q=4$, and $x+y=q$ would mean that $y=2$ also. That is impossible, because each letter represents a different digit. Therefore, $x=3$, $q=9$, and $y=6$.
5. From expression 8 find that $r+r=u$. If you substitute $2r$ for u in expression 3, $r+r+r+r=z$, you'll get $2u=z$. If you substitute z for $2u$ in expression 1, $u \times r = z$, you'll get $u \times r = 2u$. Therefore, $r=2$. Because $r+r=u$, $u=4$. Because $2u=z$, $z=8$.
6. From expression 9, $x+u=s$, you may conclude that $s=7$. Therefore, there is only one choice for the letter t. It must equal 5.

$q=4$ or 9
$r=?$
$s=?$
$t=?$
$u=?$
$v=1$
$w=0$
$x=2$ or 3
$y=?$
$z=?$

$q=9$
$r=2$
$s=?$
$t=?$
$u=4$
$v=1$
$w=0$
$x=3$
$y=6$
$z=8$

$q=9$
$r=2$
$s=7$
$t=5$
$u=4$
$v=1$
$w=0$
$x=3$
$y=6$
$z=8$

In this line of reasoning expression 5, $\frac{y}{z} = \frac{x}{u}$, was not used at all. But you can use this expression instead of some other expressions, for example, instead of expression 8. Try to figure out yourself how to do it and whether there are other possible ways of reasoning.