

FCPS Mathematics Framework



K-12 Implementation Manual

Curriculum, Instruction & Assessment – June 2017

Table of Contents

TABLE OF CONTENTS	1
SECTION 1: INTRODUCTION	2
PHILOSOPHY.....	2
SECTION 2: TIER 1 MATHEMATICS LESSON STRUCTURE.....	4
STANDARDS FOR MATHEMATICAL PRACTICE.....	5
TECHNOLOGY STATEMENT.....	7
FCPS MATHEMATICAL TEACHING STRATEGIES.....	8
SECTION 3: FCPS ESSENTIALS FOR MATHEMATICS	15
SECTION 4: SPECIAL CONSIDERATIONS	20
RIGHT TO LEARN.....	20
Student Voice.....	20
Limited English Proficient.....	21
Special Education.....	21
Gifted and Talented.....	22
REFERENCES.....	23

Section 1: Introduction

INTRODUCTION

The FCPS Math Framework is based on current research in mathematics content and best instructional practices. The content in the FCPS Math Framework reflects the skills, processes, and knowledge students in Fayette County Public Schools need to know to be competent, knowledgeable, and confident in their understanding of mathematics, and in their ability to apply this understanding in future learning experiences. The Kentucky Academic Standards and *Principles to Actions: Ensuring Mathematical Success for All* were the foundational resources used to guide the development and content of this document.

PHILOSOPHY

Learning mathematics involves a variety of skills, processes, and understandings. **We strive to educate students who not only have the understanding of mathematical concepts and procedural fluency with mathematical skills but who can model mathematical situations appropriately and construct their own learning through carefully planned classroom experiences.**

Results of longitudinal studies indicate that highly engaging student-centered math classrooms are achieving at a higher level than traditional structured classrooms (Boaler, 2008). We support student-centered classrooms that use a balanced approach to teaching mathematics, emphasizing both conceptual understanding and procedural fluency, which leads to increased mathematical proficiency for all students.

Students must have a comprehension of mathematical concepts, operations, and skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. This is accomplished through a guaranteed and viable mathematics curriculum in all Fayette County Public Schools, from early childhood through high school graduation.

Best practice further shows that students must have an ability to formulate, represent, and solve a variety of mathematical problems. Flexibility, creative thinking, and strategic competence are essential in today's information-driven, quickly changing world.

An ability to reason is essential, with a capacity for logical thought, reflection, modeling, explanation and justification. Communicating mathematically verbally and in written form is an important step in leading to greater mathematical proficiency, and all students should frequently be asked to reason, communicate, make connections, and represent mathematical ideas.

Parent involvement is a critical factor in math student achievement. We encourage continuous communication with parents about our mathematics teaching and learning and we value their support of the mathematics.

The classroom teacher of mathematics is the most important factor in weaving the strands together through daily instruction, differentiating to meet the needs of all students, and collaborating with colleagues to help all students achieve success in mathematics learning. Research has consistently

shown that achievement gaps disappear in classrooms where approximately 72% of the time students work in groups while the classroom teacher circulated the room showing students methods, helping students and asking them questions of their work (Boaler, 2008). Effective teachers integrate techniques and strategies they intentionally build on what learners already know, extend their student' skills and knowledge, and encourage inquiry, problem-solving, self-monitoring, and independent thinking.

The guidelines provided in this document are the instructional components and structures identified in research as the most effective. However, within the guideline, the specific grouping, scheduling, resources and instructional models should vary to meet the needs of the students in each individual school.

Section 2: Tier 1 Mathematics Lesson Structure

Math Routines: (10 Minutes)

Options Include:

Calendar Math
 Problem of the Day
 (Problem Solving/Problem Solver)
 Math Stretch
 -Data question (may relate to current unit of study in another subject)
 -Number of the day
 -Arithmetic Developed Daily (A.D.D.) / Math Minutes
 -What's next? (sequencing)
 -Math in real life
 _____ makes me think of...
 -Vocabulary
 -Daily Oral Math
 -Which One Doesn't Belong?
 -Estimation 180
 -My Favorite "No"
 Number Talk
 Math Journal
 3-Act Math Task
 Review from the day before, the week before, or the months before
 Introduction of a problem solving lesson
 Introduction of a Formative Assessment Lesson (TCT)

How will students demonstrate understanding?

Student-Centered Whole Group Instruction Grade Level/Course Standards (20 - 30 Minutes)

1. Setting the Stage (3 minutes)

- Use KY Academic Standards and FCPS Curriculum Map Big Ideas/Outcomes
- Ask questions to activate prior knowledge
- Review prerequisite skills for next concept
- Set instructional outcomes and assessment criteria

2. Students are Actively Engaged

Direct Instruction should not exceed:

K-2 : 5-8 minutes; Gr. 3-5 : 8-12 minutes

Gr. 6-12 : 12-15 minutes

- Students should productively struggle, explore, persevere, share, justify, and compare
- Use concrete examples and move to pictorial, then toward abstract/symbolic

Examples: literature, vocabulary, journals, graphic organizers, technology, math talk manipulatives, individual white boards

- Use Guided Practice
- Use Independent Practice
- Use Through Course Tasks (Formative Assessment Lessons - FALs)
- Use Structured Group Work

3. Review & Discuss (3-5 minutes)

Review instructional outcome and discuss assignment for small groups

How will students demonstrate understanding?

Tier 1 Mathematics
Standards for Math Practice should
 be intentionally integrated
(60 - 90 Minutes)
Recommended

Small Group Instruction/Guided Math Differentiated Tier 1 Instruction (20 - 30 Minutes)

While students are working at seats or stations, teacher pulls small groups for guided lessons. Student work is organized/ differentiated based on student needs.

- **Guided Math** – teacher works with 2-3 guided math groups daily, each group approximately 15-20 minutes.
- **Intervention/Supplemental** – teacher works with additional students as needed, either 1-on-1 or in small group (not to exceed 3-4 students) approximately 15-20 minutes.

Closing/Re-Cap (5 Minutes)

This could be written or oral:

- using math discussion with high-level questioning
- student white boards
- Kahoot
- journal writing
- exit slips
- **re-visit instructional outcome and assessment criteria**

These strategies should vary from day to day.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These research-based practices are founded on important “processes and proficiencies” with longstanding importance in mathematics education.

STANDARDS FOR MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

1. Make sense of problems and persevere in solving them.

- Identify a problem and make a plan
- Explain to themselves the meaning of a problem.
- Use a variety of appropriate strategies to solve problems.
- Explain relationships between equations, verbal descriptions, tables and graphs.
- Draw diagrams of important features and relationships.
- Monitor and evaluate their progress and change course if necessary.
- Check their answers to problems using a different method.
- Justify their process and outcome.
- Identify relationships between different approaches of others.

2. Reason abstractly and quantitatively.

- Make sense of numbers and their relationships to problems.
- Use reasoning to represent how to solve a problem versus computation alone.
- Know and flexibly use different properties of operations.
- Use different objects to represent a problem.

3. Construct viable arguments and critique the reasoning of others.

- Understand and use situations to construct arguments.
- Make predictions and use data to support and validate their predictions.

4. Model with mathematics.

- Apply the mathematics they know to solve everyday problems.
- Identify important information in a situation.
- Use tools, such as tables and graphs, to express the solution.
- Analyze solutions to draw conclusions.
- Interpret results in the context of the situation.
- Reflect on whether the results make sense and revise if necessary.

5. Use appropriate tools strategically.

- Consider available tools and resources when solving a mathematical problem.
- Are familiar with grade appropriate tools.
- Use estimation to detect possible errors.
- Use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

- Communicate precisely with others.
- Use clear definitions in discussions with others.
- State the meaning of the symbols they choose.
- Specify units of measure and label axes.
- Calculate accurately and efficiently.

7. Look for and make use of structure.

- Look to find a pattern or structure.
- Check for accuracy of the pattern and revise as necessary.
- See that complex problems can be broken down to make solving easier.
- Reflect on the problem as a whole.

8. Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated.
- Look both for general methods and for shortcuts.
- Maintain oversight of the process, while attending to the details.
- Continually evaluate the reasonableness of intermediate results.

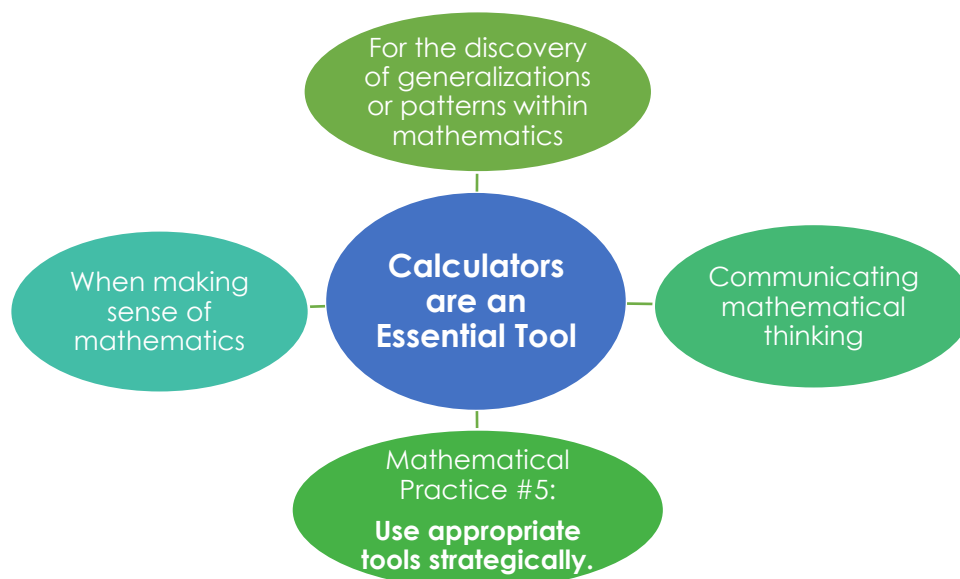
TECHNOLOGY STATEMENT

An effective math classroom integrates technology as an essential resource (NCTM, 2014). The vast array of technologies available to teachers should be integrated as a modern tool of instruction to make mathematics more meaningful and accessible for all students.

- It is essential teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Computers, tablets, smartphones, and calculators are all tools that support reasoning and problem solving.
- Effective teachers optimize the use of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they provide greater access to mathematics for all students.
- Technological tools include those that are both content specific and content neutral. In math education, content-specific technologies include computer algebra systems; dynamic geometry environments; interactive apps; handheld computation, data collection, and analysis devices; and computer-based applications. Content-neutral technologies include communication and collaboration tools, word processing, presentation software and Web-based digital media. Some examples may include Gizmos, NCTM Illuminations and Probability Software. Virtual manipulatives help students to visualize relationships.
- Students' sharing of work can occur through Web-based platforms. Students might use text messaging, cloud-based shared documents, virtual blogs or wikis to work and collaborate.

Calculators:

- ✓ Research shows calculator use does not contribute to negative skill development, but instead, enhances the understanding of math concepts (Ronau, 2001).
- ✓ Students should be taught when to use a calculator and when mental computing is more effective or appropriate. Choosing the right "tool" is part of an effective problem-solving process.
- ✓ Calculators should be used when they enhance student learning.
- ✓ Calculators should be used when the computations become so cumbersome that they are an obstacle to learning higher-level concepts.



FCPS Mathematical Teaching Strategies

The teaching of mathematics is complex and requires teachers to have a deep understanding of the mathematical knowledge they teach (Ball, 2008). It is essential for teachers to have a clear view of how student learning of mathematics develops and progresses across grades and in ways that effectively develop mathematics learning for all students (Daro, 2011).

The five FCPS Mathematical Teaching Strategies are adapted from *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) which lists teaching practices that encourage effective teaching and learning.

- 1 • Establish and communicate mathematics learning **outcomes** to focus learning throughout the lesson (pg. 9)
- 2 • Implement tasks that **promote reasoning and problem solving** using **mathematical representations** (pg.10)
- 3 • Facilitate meaningful mathematical **discourse** and assign **purposeful questions** (pgs.11-12)
- 4 • Build **procedural fluency** from conceptual understanding (pg.13)
- 5 • Support **productive struggle** in learning mathematics and use **evidence** of student thinking (pg.14)

1. ESTABLISH AND COMMUNICATE MATHEMATICS **LEARNING OUTCOMES** TO FOCUS LEARNING THROUGHOUT THE LESSON

Effective teaching of mathematics:

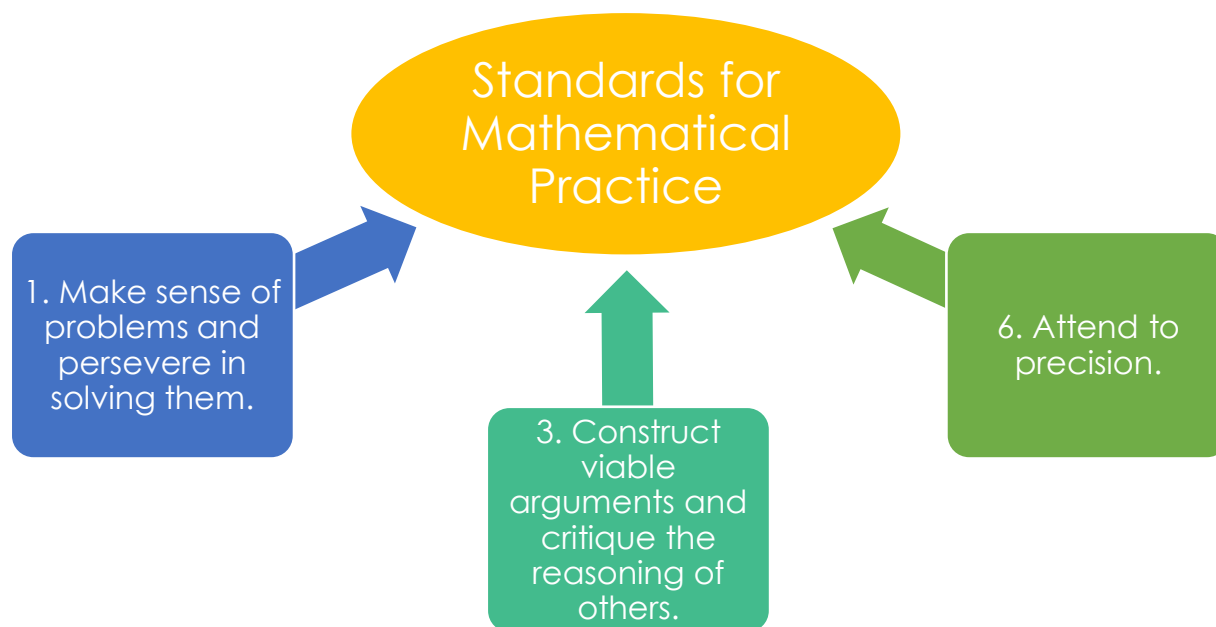
- establishes clear learning outcomes for the mathematics that students are learning
- situates instructional outcomes within learning progressions and clarified goals within an instructional unit
- uses the instructional outcomes to guide teacher's instructional decisions as well as students' monitoring of their own progress

Teacher Actions	Student Actions
Establish clear learning outcomes that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.	Engage in discussions of the mathematical purpose and instructional outcomes related to their current work in the mathematical classroom (What are we learning? Why are we learning it?)
Identify how instructional outcomes fit within a mathematics learning progression.	Use the instructional outcome to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices.
Discuss and refer to the mathematical purpose and instructional outcome of a lesson during instruction to ensure that students understand how the current work contributes to their learning.	Connect their current work with the mathematics that they studied previously and see where the mathematics is going.
Use the mathematics instructional outcomes to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Assess and monitor toward the mathematics learning outcomes.

Fig. 1: Math Goals NCTM, 2014, p.16

Important Instructional Considerations:

- Communicate what students will understand based on instruction
- Identify mathematical practices that students are learning to use more proficiently
- Should not just be reiteration of standards but should be linked to curriculum and learning



2. IMPLEMENT TASKS THAT PROMOTE REASONING AND PROBLEM SOLVING USING MATHEMATICAL REPRESENTATIONS

Effective teaching of mathematics:

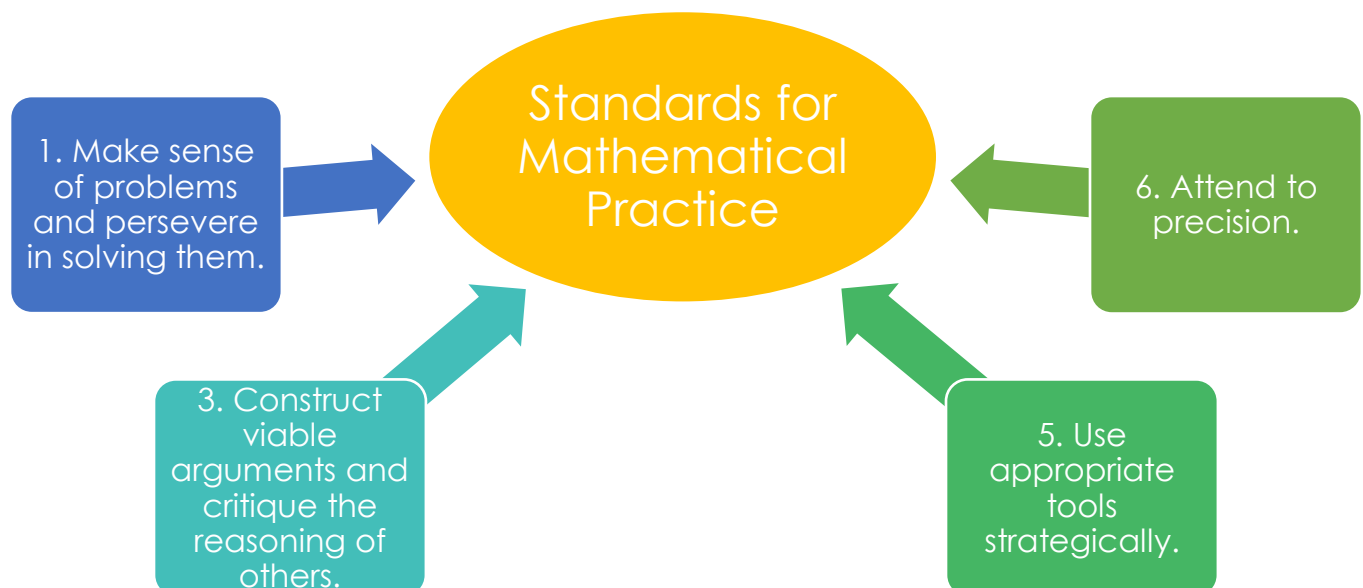
- engages students in solving and discussing tasks that promote mathematical reasoning and problem solving
- allows for multiple entry points and varied solution strategies
- engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tool for problem solving

Teacher Actions	Student Actions
Motivate students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.	Persevere in exploring and reasoning through tasks.
Select tasks that provide multiple entry points through the use of varied tools and representations.	Take responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.
Pose tasks on a regular basis that require a high level of cognitive demand. Support students in exploring tasks without taking over student thinking.	Use tools and representations as needed to support their thinking and problem solving.
Encourage students to use varied approaches and strategies to make sense of and solve tasks.	Accept and expect that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.
Design ways to elicit and assess students' abilities to use representations meaningfully.	Connect mathematical ideas and concepts to real-world situations.

Fig. 2: Tasks NCTM, 2014, pgs. 24 and 29

Important Instructional Considerations:

- Provide rich, open ended tasks that allow for multiple solutions
- Model and use visual supports
- Help students to see the connection between the different representations



3. FACILITATE MEANINGFUL MATHEMATICAL DISCOURSE AND ASSIGN PURPOSEFUL QUESTIONS

Effective teaching of mathematics:

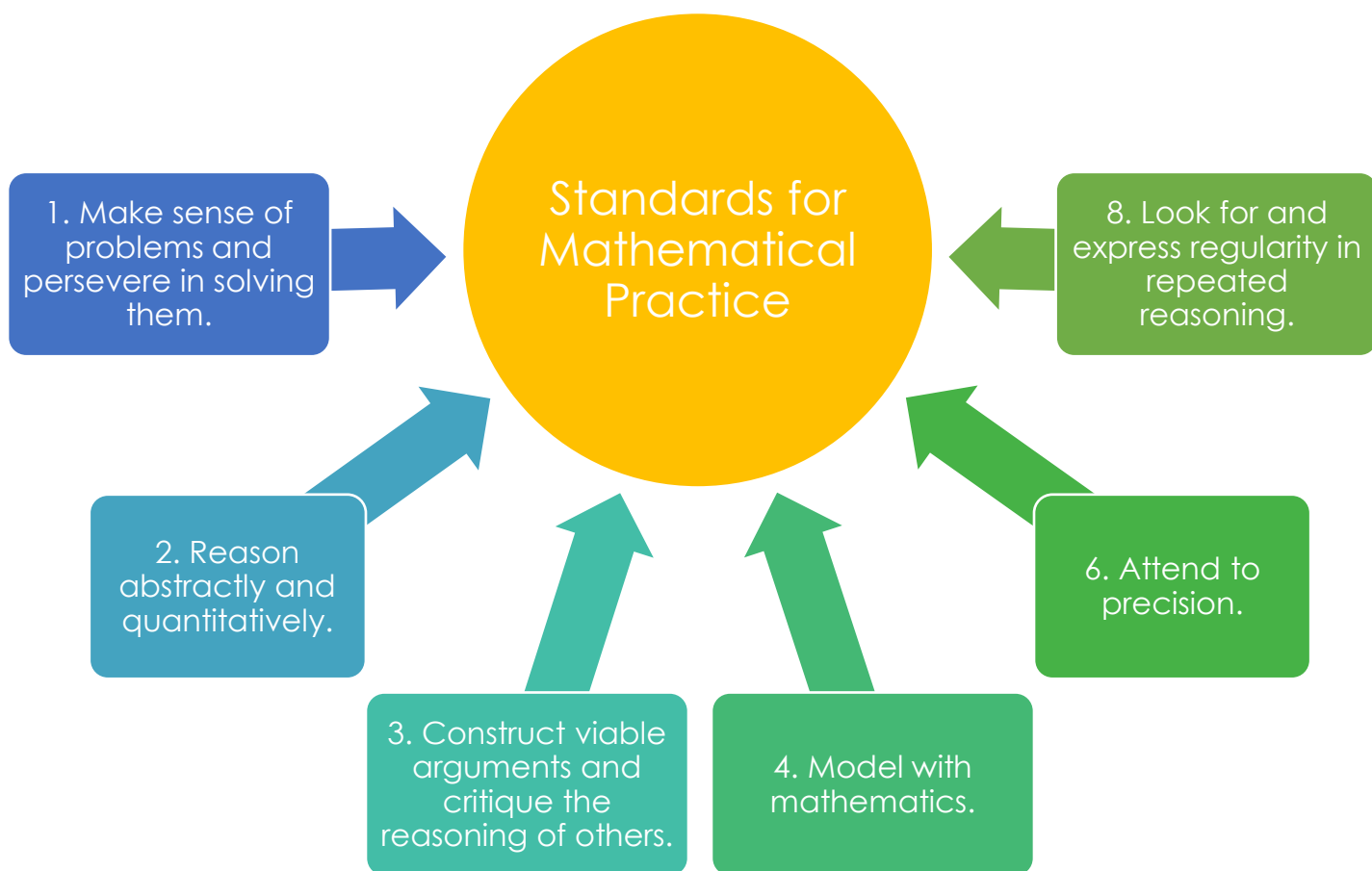
- facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments

Teacher Actions	Student Actions
Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	Present and explain ideas, reasoning, and representations to one another in pairs, small group, and whole-class discourse.
Scaffold student approaches and solution strategies for whole-class analysis and discussion.	Use examples to support or counterexamples to refute conclusions and solutions.
Facilitate discourse among students by having them justify and explain their reasoning for their answer and approach.	Seek to understand the approaches used by peers by asking clarifying questions, try out others' strategies, and describe the approaches used by others. Listen carefully critique the reasoning of peers
Ensure progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Identify how approaches to solving a task are the same and how they are different.
Ask questions that go beyond gathering information. The questions should probe students' thinking and require explanation and justification.	Think carefully about how to present their responses to questions clearly, without rushing to respond quickly.

Fig. 3: Discourse NCTM, 2014, pgs. 35 and 41

Important Instructional Considerations

Teacher Role	<ul style="list-style-type: none"> • Students carry the conversation themselves. • Teacher only guides from the periphery of the conversation. • Teacher waits for the students to clarify thinking of others.
Questioning	<ul style="list-style-type: none"> • Student to student talk is student initiated. • Students ask questions and listen to responses. • Many questions ask why and call for justification. • Teacher questions may still guide discourse.
Explaining Mathematical Reasoning	<ul style="list-style-type: none"> • Teacher follows student explanations closely. • Teacher asks students to contrast strategies. • Students defend and justify their answers with little prompting from the teacher.
Mathematical Representations	<ul style="list-style-type: none"> • Students follow and help shape the descriptions of others math thinking through math representations and may suggest edits in others representation.
Building student responsibility within the community	<ul style="list-style-type: none"> • Students believe that they are math leaders and can help shape the thinking of others. • They help shape others math thinking in supportive, collegial ways and accept the same support from others.



Student Materials:

Materials will be dependent on the lesson that you are building depending on representations, communication tools, justifying, and responsive classroom techniques.

A variety of manipulatives should be available to students to promote understanding and sense making of the mathematics. (This is not a limited list, but suggestions of tools to have available.)

calculators	number line	base ten blocks	algebra tiles
dice	100s charts	measurement tools	pattern blocks
clocks	graph paper	five and ten frames	Cuisenaire rods
diagrams	number bonds	1 in. or cm tiles/cubes	attribute shapes
tangrams	linking cubes	two color counters	3D shapes
geoboards	fraction pieces	place value chips	money sets
rekenreks	counters	dry erase boards	pentominoes
	dominoes	calculator diagrams	

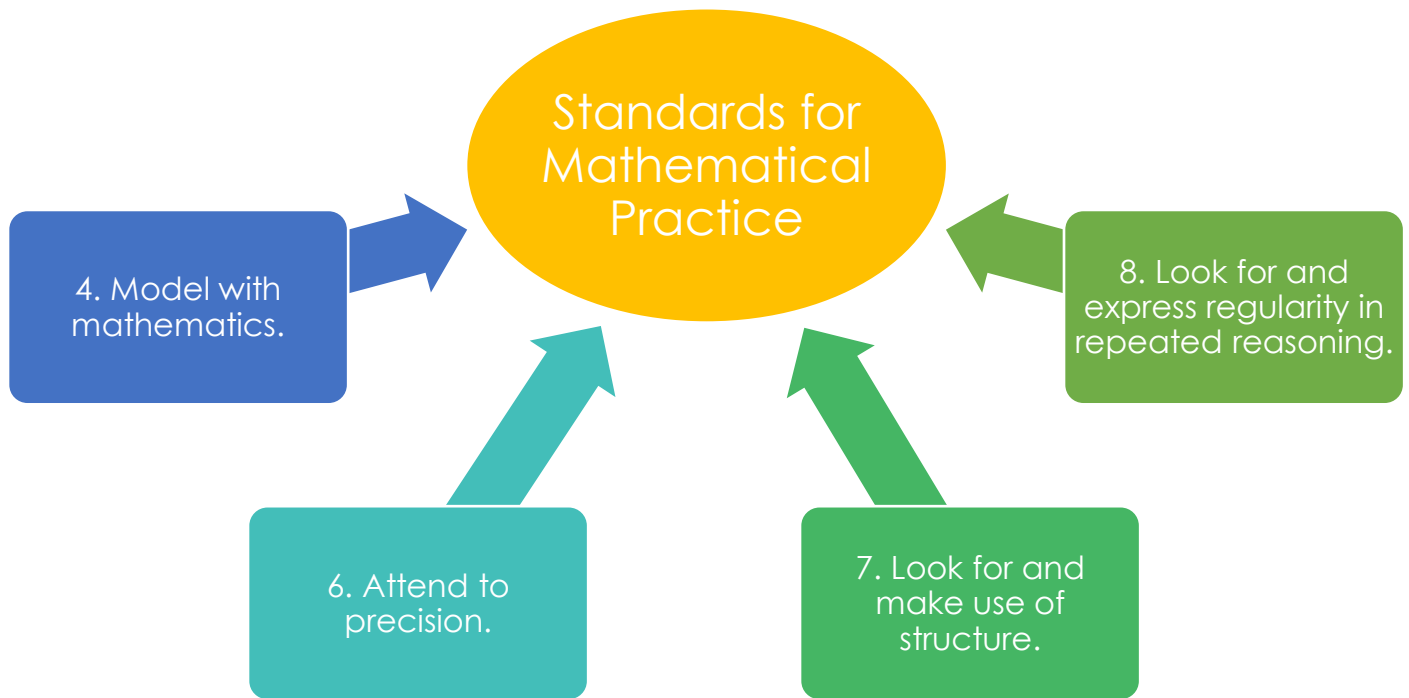
4. BUILD **PROCEDURAL FLUENCY** FROM CONCEPTUAL UNDERSTANDING.

Effective teaching of mathematics promotes students to be able to:

- Build fluency with procedures on a foundation of conceptual understanding
- Become skillful in flexibly using procedures as they solve contextual and mathematical problems

Teacher Actions	Student Actions
Acknowledge the importance of both conceptual understanding and procedural fluency.	Demonstrate the ability to choose flexibly among methods and strategies to solve problems.
Provide students opportunities to use their own reasoning strategies and methods for solving problems.	Demonstrate knowledge by practicing on a moderate number of carefully selected problems once they have a strong conceptual foundation and can explain the use of the strategy.
Ask students to discuss and explain why the procedures that they are using work to solve particular problems.	Access procedures that they can use with understanding on a broad range of problems.
Connect student generated strategies and methods to more efficient procedures as appropriate.	Know which procedure is appropriate and most productive in a given situation.
Use visual models to support students' understanding of general methods.	Are able to explain their approaches, and are able to produce accurate answers efficiently.
Provide students with opportunities for continuous practice of procedures.	
Provide students time to practice math facts.	

Fig. 4: Fluency NCTM, 2014, pg.47



5. SUPPORT **PRODUCTIVE STRUGGLE** IN LEARNING MATHEMATICS AND USE **EVIDENCE** OF STUDENT THINKING

Effective teaching of mathematics:

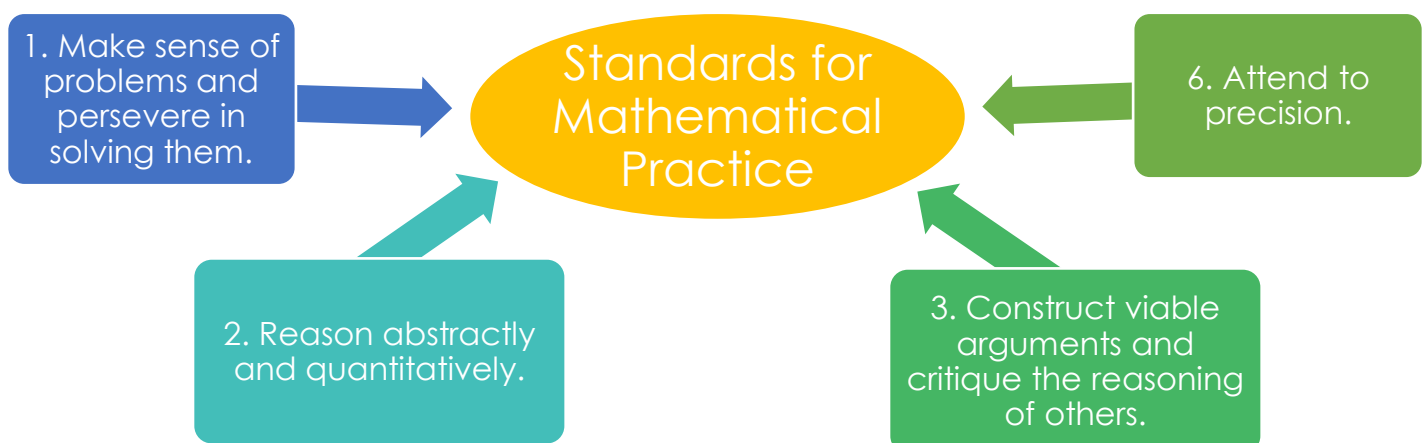
- Consistently provides students with opportunities and supports to engage in productive struggle
- Opportunities for delving more deeply into understanding the mathematical ideas
- Able to apply their learning to new problem situations
- Uses evidence of student thinking to assess progress toward understanding
- Uses evidence to adjust instruction continually in ways that support and extend

Teacher Actions	Student Actions
Anticipate what students might struggle with during a lesson and being prepared to support them productively through the struggle.	Struggle at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
Give students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.	Ask questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
Help students realize that confusion and errors are a natural part of learning, by facilitating discussion on mistakes, misconceptions, and struggles.	Persevere in solving problems and realizing that it is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
Praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Help one another without telling their classmates what the answer is or how to solve the problem.
Elicit and gather evidence of student understanding at strategic points during instruction.	Reflect on mistakes and misconceptions to improve their mathematical understanding.
Make in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.	Assess and monitor their own progress toward mathematics learning goals and identifying areas in which they need to improve.

Fig. 5: *Student Thinking* NCTM, 2014, pgs. 52 and 56

Important Instructional Considerations:

- Consider and discuss growth mindset and the importance of mistakes for learning
- Students question and critique the reasoning of their peers
- Students reflect on their own understanding
- Students have access to tools that will support their thinking processes



Section 3: FCPS – Essentials Skills for Mathematics

This “Big Rocks” document, created by Fayette County teachers, is a list of essential learning for mastery of grade-level content. It is *not* to be interpreted as a complete list of all topics to be taught. Content vocabulary should be an emphasis at ALL grade levels. *FCPS Mathematical Teaching Strategies* (pgs. 8-14) should be used for instruction with Standards for Mathematical Practice embedded throughout. The tool is aligned with Advanced Ed Standards *Articulating Expected Mastery of Skills and Standards* as well as *High Expectations for Students*.

Kindergarten:

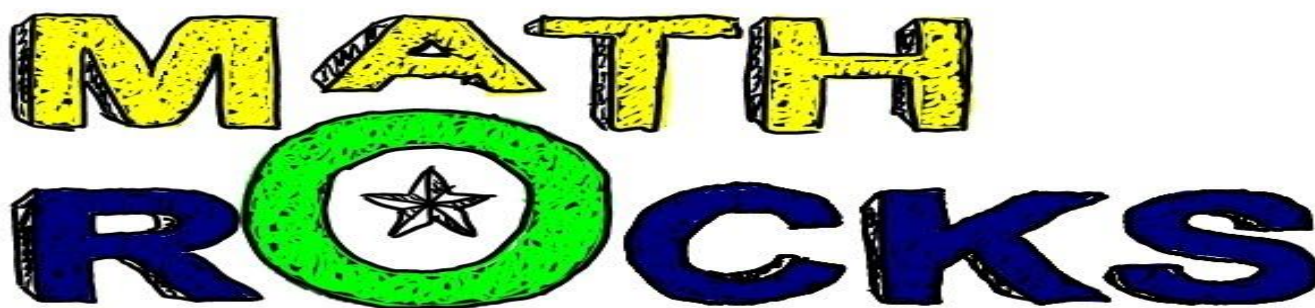
1. Know number names and the count sequence. Count (forward) to 100 by 1's and 10's and count forward or backward from a given number within 30.
2. Identify and write numbers 1 to 20.
3. Count to tell the number of objects to 20. Count out objects to represent a number to 20.
4. Compare numbers (greater than, less than, or equal to) another number within 20.
5. Tell the **next** number that is one more (or one larger) and the number **before** that is one less (or one fewer), within numbers to 20.
6. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. **Required fluency: Add and subtract within 5 using mental math.**
7. Work with numbers 11–19 to gain foundations for place value.
8. Add to a given number to make 10 and record answer with drawings and equations.
9. Describe and compare measureable attributes.
10. Classify objects and count the number of objects in categories.
11. Identify, describe, create, compare, and compose 2D and 3D shapes (square, circle, rectangle, triangle, hexagon, cube, cone, cylinder, and sphere).

1st Grade:

1. Count to 120 starting at any given number.
2. Read, write, and represent any given numeral between 0 and 120.
3. Compare numbers (greater than, less than, or equal to) another number within 120.
4. Represent and solve problems involving addition and subtraction.
5. Understand and apply properties of operations and the relationship between addition and subtraction to add and subtract within 20. **Required fluency: Add and subtract within 10.**
6. Work with addition and subtraction equations within 100, using different strategies, including models. Work with 3 types of addition and subtraction problems: result unknown, change unknown, and start unknown. Model add-to, take-from, put-together, take-apart.
7. Understand the position of each digit in a number impacts the quantity of a number.
8. Use place value understanding and properties of operations to add and subtract.
9. Measure lengths indirectly and by iterating length units. Order 3 objects by length.
10. Tell and write time, to the hour and half hour, using *both* analog and digital clocks.
11. Represent data in a chart or table and interpret data (up to 3 categories).
12. Recognize and understand patterns in a 0-99 chart and a hundreds chart.
13. Reason with shapes and their attributes (sort, compare, compose, decompose, and partition into equal parts (halves, fourths)).

2nd Grade:

1. Count to 1000 by 1s, 5s, 10s, and 100s.
2. Read, model, and write numbers to 1000 using base-ten numerals, number names, diagrams, number sentences, and expanded form.
3. Compare two 3-digit numbers based on the values of the hundreds, tens, and ones digits.
4. Use place value understanding and properties of operation to represent and solve problems involving addition and subtraction.
5. **Required fluency: Recall from memory all single-digit sums and differences within 20.**
6. **Required fluency: Fluently add and subtract 2-digit numbers within 100.**
7. Work with equal groups of objects to gain foundations for multiplication.
8. Explain the value of each digit in a 3-digit number including zeros in the tens or ones place.
 - a. Understand the difference between place and value.
9. Measure and estimate lengths in standard units. Relate addition and subtraction to length.
10. Count and solve word problems with pennies, nickels, dimes, quarters, bills, and symbols.
11. Use charts, tables, and surveys to collect and graph data on a bar graph or pictograph.
12. Describe plane figures (sides, corners, angles) and solid figures (faces, edges, vertices).
13. Identify and represent fractional parts of a whole (halves, thirds, fourths).

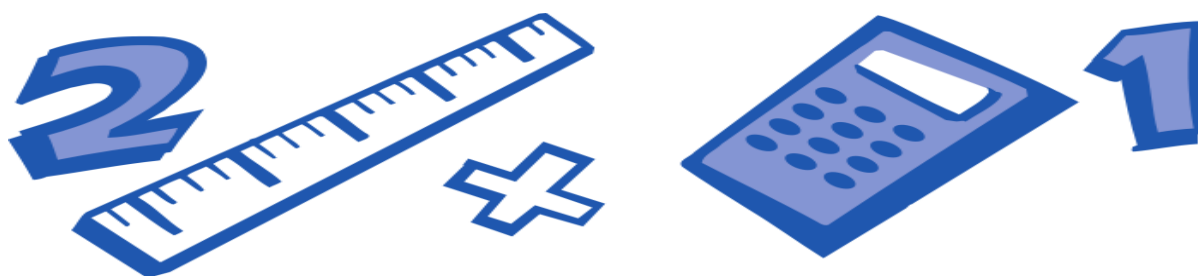


3rd Grade:

1. Represent and solve problems involving multiplication and division. **Required fluency: Single-digit products and quotients from memory by end of Grade 3.**
2. Understand properties of multiplication *and the relationship* between multiplication and division. **Required fluency: Multiply and divide within 100.**
3. Solve problems involving the four operations, and identify and explain patterns in arithmetic.
4. Use place value understanding and properties of operations to perform multi-digit arithmetic. **Required fluency: Add and subtract within 1000.**
5. Develop understanding of fractions as numbers (beginning with unit fractions).
6. Represent fractions on a number line.
7. Compare fractions of denominators 2,3,4,6, and 8 using a visual fraction model.
8. Generate equivalent fractions of denominators 2,3,4,6, and 8.
9. Solve problems involving measurement (nearest $\frac{1}{4}$ inch), elapsed time, liquid volumes, and masses of objects.
10. Collect, represent, and interpret data on line plots.
11. Understand concept of area. Relate area to multiplication and to addition through use of arrays.
12. Solve problems involving perimeters of polygons. Distinguish between perimeter and area.
13. Reason with two-dimensional shapes and their attributes (sort, compare, classify, describe examples, describe nonexamples).

4th Grade:

1. Generalize and use place value understanding and properties of operations to perform multi-digit arithmetic. **Required fluency: Add and subtract within 1,000,000.**
2. Use the four operations with whole numbers to solve problems, including word problems.
3. Multiply 4x1 and 2x2 numbers and find quotients and remainders with up to 4-digit dividends and 1-digit divisors.
4. Find factors and multiples of a number.
5. *Extend 3rd grade understanding of fraction equivalence and ordering to include denominators of 5, 10, 12, and 100 using visual fraction models.*
6. Add and subtract proper fractions, improper fractions, and mixed numbers with **like denominators**, using visual fraction models and equations. Build fractions from unit fractions.
7. Multiply a fraction by a whole number using models and equations.
8. Understand decimal notation. Compare two decimals to hundredths.
9. Locate fractions and decimals on a number line.
10. Solve problems involving measurement *and conversion* of measurements from a larger unit to a smaller unit (including ALL standard measures, ALL metric measures, money, and time).
11. Represent and interpret data.
12. Use area and perimeter to solve for unknown measures.
13. Understand concepts of angle and measure angles with a protractor. Draw and identify lines and angles; classify shapes by properties of their lines and angles (including parallel, perpendicular, and symmetry).
14. Generate both number and shape patterns that follow a rule and analyze patterns.



5th Grade:

1. Understand the place value system, including decimals to hundredths.
2. Perform all four operations with multi-digit whole numbers including order of operations (including parentheses and brackets). **Required fluency: Multi-digit multiplication (3- or 4-digit number multiplied by a 2- or 3-digit number).**
3. Add, subtract, multiply, divide decimals to hundredths.
4. Compare decimals to the thousandths place.
5. Round decimals to the thousandths place.
6. Add and subtract fractions and mixed numerals (including **unlike** denominators).
7. Multiply fractions and mixed numerals. Divide fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).
8. Find area of a rectangle with fractional side lengths.
9. Convert like measurement units within a given measurement system.
10. Understand concepts of volume and relate volume to multiplication and to addition.
11. Graph points on the coordinate plane to solve real-world and mathematical problems.
12. Classify two-dimensional figures into categories based on their properties.
13. Generate 2 numerical patterns given two rules.

6th Grade:

1. Extend understanding of fractions, decimals, and percents.
2. Understand and use ratios, ratio reasoning and unit rates.
3. Solve algebraic expressions.
4. Solve and interpret 1-step equations and 1-step inequalities.
5. Construct, analyze and interpret data in a variety of graphical manners (number line, line plot, dot plot, histogram, box plot (box and whiskers)); compute mean, median, mode and range.
6. Find the area of complex 2-D figures (including composing or decomposing figures), review volume of 3-D figures with fractional side lengths, and calculate the surface area of 3-D figures.
7. Represent and understand integers and position on both horizontal and vertical number lines including ordering, comparing, and absolute value.
8. Extend understanding of the coordinate plane to all four quadrants.

7th Grade:

1. Analyze and use proportions and proportional reasoning including scale drawings.
2. Represent proportional relationships with the constant of proportionality in tables, graphs, equations and verbal descriptions.
3. Solve and apply percent problems including tax, gratuities, discount, simple interest and percent of change.
4. Perform operations on rational numbers including integers and positive/negative fractions & decimals.
5. Determine and analyze probabilities by constructing sample space and conducting sample and conducting experiments.
6. Solve problems involving area and circumference of circles.
7. Solve equations for unknown angle measures including complementary, supplementary, vertical and adjacent angles.
8. Use central tendency and variability to compare two sets of data.
9. Solve and interpret multi-step equations and inequalities.

8th Grade:

1. Work with irrational numbers, radicals and integer exponents.
2. Graph linear equations and extend understanding of slope as the rate of change.
3. Solve multi-step equations including those with variables on both sides, the distributive property, and combining like terms.
4. Solve systems of two linear equations in two variables algebraically and estimate solutions graphically (both by hand and on a graphing calculator).
5. Investigate and interpret patterns of association in bivariate data using scatterplots and lines of fit.
6. Define, evaluate and compare functions using tables, graphs, equations, and verbal descriptions.
7. Understand and apply the Pythagorean Theorem.
8. Work with transformations in a coordinate plane.
9. Work with parallel lines cut by a transversal.

Algebra 1:

1. Solve multi-step equations and inequalities in one variable and represent the solution on a number line.
2. Write and graph linear equations in two variables that model real world situations.
3. Solve systems of equations by multiple methods and interpret their solutions in real world context.
4. Use function notation to perform arithmetic operations; find the domain and range of functions.
5. Perform arithmetic operations on polynomials.
6. Use rational and irrational numbers in the appropriate context of a problem.
7. Factor quadratic functions; Solve and graph quadratic equations using multiple methods.
8. Summarize, represent and interpret one or two variable data.

Geometry:

1. Use logic and proof to reason mathematically; make conjectures about, points, lines, angles, planes, polygons and other geometric figures.
2. Use various methods to prove figures are congruent or similar.
3. Classify polygons by their properties and use those properties to solve problems (parallel, perpendicular, angle relationships, triangles, etc.).
4. Use coordinate geometry (midpoint, distance, circles, parabolas) to analyze figures and solve problems.
5. Use properties of circles to solve problems involving chords, secants, tangents, inscribed angle, arcs, etc.
6. Introduce basic concepts of trigonometry including Pythagorean Theorem, sine, cosine, tangent, 45-45-90 and 30-60-90 triangles and use trig ratios to solve real world problems.
7. Use surface area and volume to analyze three-dimensional figures including cross sections and ratios of perimeter, area and volume.

Algebra 2:

1. Solve multistep linear equations and compound inequalities involving absolute value and graph the solution on a number line, when applicable.
2. Solve systems of equations and inequalities using multiple methods as appropriate.
3. Solve and graph quadratic equations using real and complex numbers; use the discriminant to determine the number and types of solutions; find domain and range.
4. Identify and graph conic sections.
5. Factor, solve, and graph polynomial equations. Determine the number and type of zeros for a polynomial; use maximums, minimums, zeros, intercepts to graph a polynomial; find domain range.
6. Use operations on radical expressions and solve equations. Include rational and negative exponents, nth roots and rationalizing denominators.
7. Use logarithms to simplify expressions and solve equations.
8. Perform operations on rational expressions and solve rational equations.
9. Expand knowledge of trigonometry to include all six trig functions, the unit circle, radian measure, Law of Sines and Law of Cosines, graphs of trigonometric functions including amplitude and period.
10. Use counting principle to find the number of ways an event can happen and find the probability of that event.
11. Find the nth term in an arithmetic or geometric sequence and find the sum of a series.

Section 4: Special Considerations

Each student has the right to learn

It is our collective responsibility as an education community to make certain each child receives a high quality, challenging education designed to maximize potential, as well as an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. To be successful, education must be committed to serving the learning needs of students from various social, economic, cultural, linguistic, and developmental backgrounds. For all students to have a guaranteed right to learn, schooling must be equitable. These components include a call to provide **ALL** students with the following:

Equitable access to math resources and facilities.

Instruction, in all areas of mathematics, including Math Practice Standards, tailored to their needs.

Math curriculum that is rigorous and relevant.

Educators who are culturally sensitive and respectful.

Interactions with staff and other students that are positive and encouraging in a growth mindset atmosphere.

Assessment that is varied to give each student the opportunity to demonstrate learning in math.

Student Voice

Student Voice is an opportunity for teachers to receive feedback from students related to their learning experiences. It provides formative evidences for teacher reflection in order to improve classroom culture and instructional practices.

The following criteria represents student expectations for a mathematics classroom as collected in feedback sessions from Fayette County students:

real-world examples	engaging activities	hands-on learning	technology
differentiated assignments	immediate feedback	study guides and supports	guided instruction

Limited English Proficient

Each school in FCPS provides EL instructional services to students designated as LEP (Limited English Proficient). Delivery of services may vary at the school or educational level. Some school programs may use a combination of EL instructional program models, depending on staffing and other available school resources.

Mathematics teachers must attend to all students, including those who speak a first language other than English or have related cultural differences, and ensure that all have access and opportunities to learn mathematics and to reveal what they know. Students who are not fluent in English can learn math at grade level or beyond at the same time they are learning English when appropriate strategies are used (NCTM 2014, 63).

Expanded learning opportunities and instructional accommodations should be available to English language learners (ELLs) who need them to develop mathematical understanding and proficiency.

The following instructional program models are used in FCPS to teach ELs.

CBE

Content-Based ESL

LEP students are specifically grouped to receive instruction in English to develop language skills while preparing them to study grade-level math material in English.

POE

Pull-Out ESL

LEP students in elementary schools are pulled from the general classrooms for a portion of the day to pre-teach, teach or reteach English language skills and /or mathematics content covered by the general education classroom teacher. Instruction is in English.

SEN

Structured English Immersion

LEP students remain in their mainstream math classrooms receiving English language assistance from push-in, collaboration, consultation and various other modifications/differentiation by classroom teacher. Strategies include the pre-teaching of targeted math vocabulary and reading strategy instruction to support solving word problems.

Special Education

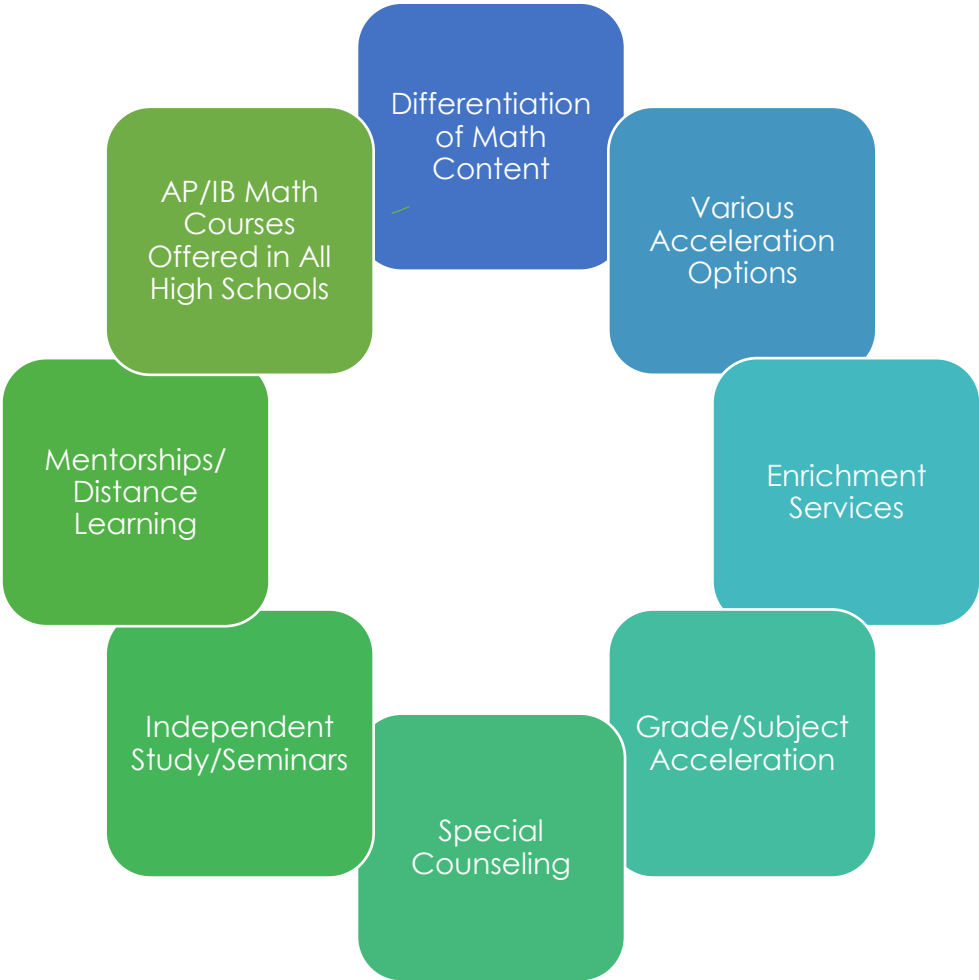
Students receiving special education services should be provided mathematical accommodations according to their Individualized Education Program (IEP) or 504 Plan to access grade level mathematics curriculum successfully. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students (NCTM 2000, 12).

Gifted and Talented

The goal of the Fayette County Public Schools (FCPS) Gifted Education program is to provide high quality differentiated instruction supported by a challenging and rigorous curriculum (Product, Process, Content, Learning Environment). Specifically, the Gifted and Talented (GT) program is designed to meet the needs and interests of all students demonstrating potential in the five designated areas of gifted education. All schools must provide services for exceptional/gifted children in general intellectual ability, specific academic aptitude (language arts, **math**, science, and social studies), creativity, leadership and visual/performing arts (drama, dance, music, visual arts).

Students with exceptional mathematical promise must be engaged in enriching learning opportunities during and outside the school day to allow them to pursue their interests, develop their talent, and maintain their passion for mathematics. Such opportunities must be open to a wide range of students who express a higher degree of interest in mathematics, not just to those who are identified through traditional assessment instruments.

Math teachers must organize, consult, and collaborate with GT teachers to ensure multiple service delivery options are delivered to all students selected for the Primary Talent Pool and GT in all five gifted areas. FCPS Service Delivery Options (as listed in the gifted regulation 704 KAR 3:285) include:



Grouping is mandated by state gifted regulation for PTP and GT. Grouping for instructional purposes in mathematics *may include* flexible, clustered, like ability cooperative, and resource groups.

References

- Ball, Deborah. "Content Knowledge for Teaching: What Makes It Special?" *Journal of Teacher Education* 59, no.5 (2008): 389-407.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. New York, NY: Jossey-Bass.
- Boaler, J. "Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School." *Teachers College Record* 110.3 (2008): 608-45. Print.
- Daro, Phil, Frederic A. Mosher, and Tom Corcoran. *Learning Trajectories in Mathematics: A Foundation for Standards, Curriculum, Assessment and Instruction*. Philadelphia: Consortium for Policy Research in Education, 2011.
- Illustrative Mathematics. (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K–5*. Tucson, AZ.
- Lester, F. K. (2007). *Second handbook of research on mathematics teaching and learning*. Charlotte, N.C.: Information Age.
- National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.
- National Governors Association (NGA) Center for Best Practices & Council of Chief State School Officers (CCSSO). (2010). *Common Core State Standards for Mathematics*. Washington, DC.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM
- Ronau, Robert N., Christopher Rakes, Sarah Bush, Shannon Driskell, Margaret Niess, and David Pugalee. *Using Calculators for Teaching and Learning Mathematics*. NCTM Research Brief. Reston, VA.: National Council of Teachers of Mathematics, 2000.
- Van de Walle et al. (2012). *Elementary and Middle School Mathematics: Teaching Developmentally* custom edition: Boston, MA : Allyn & Bacon.